Vol. 52, No. 4 DUKE MATHEMATICAL JOURNAL © December 1985

VARIETIES WITH SMALL DUAL VARIETIES II

LAWRENCE EIN

§0. Introduction. Let X be an irreducible n dimensional closed subvariety of $\mathbf{P}^N$. Let C_X be the conormal variety of X ([15]). There is a natural projection map $p_2: C_X \to \mathbf{P}^{N*}$. $p_2(C_X) = X^*$ is called the dual variety of X. X is said to be reflexive if the map $p_2: C_X \to X^*$ is separable ([15]). If X is reflexive, then C_X is also the conormal variety of X^* ([13]). In particular, $(X^*)^* = X$.

In the following we shall assume that X is a nonlinear reflexive projective n-fold in $\mathbf{P}^N$. A. Landman defined the defect of X to be $\operatorname{def}(X) = N - 1 - \dim X^*$. For most examples X^* is a hypersurface and hence $\operatorname{def}(X) = 0$. The purpose of this paper is to investigate those varieties with positive defect. Assume that $\operatorname{def}(X) = k > 0$. Let H be a general tangent hyperplane of X. The contact locus of H with X is a k-plane L in X. In [4] we show that $N_{L/X}$, the normal sheaf of L in X, is isomorphic to $N^*_{L/X} \otimes \mathcal{O}_L(1)$. If T is a line in L, then $N_{L/X}|_T = ((n-k)/2)\mathcal{O}_T \oplus ((n-k)/2)\mathcal{O}_T(1)$. In this paper we shall investigate the deformations of L. Zak and Landman proved that $\operatorname{def}(X) \leqslant n - 2$. In 3.1, we show that if $\operatorname{def}(X) = n - 2$ ($n \geqslant 3$), then X is a scroll. In §4, we show that if $\operatorname{def}(X) = k \geqslant n/2$, then X is a $\mathbf{P}^{(n+k)/2}$-bundle over a $(n-k)/2$-fold.

Mumford showed that if X is the Plücker embedding of $G(2, 2m+1)$ ($m \geqslant 2$), then $\operatorname{def}(X) = 2$ ([22]). A. Landman and M. Reid observed that if X is a $\mathbf{P}^m$-bundle over a $(n-m)$-fold such that the fibers are embedded linearly, then $\operatorname{def}(X) \geqslant 2m - n$ when $2m > n$.

In §5, we show that if X is a n-fold with $n \leqslant 6$ and $\operatorname{def}(X) = k > 0$, then X is one of the following varieties:

(a) X is the Plücker embedding of $G(2,5)$.
(b) X is a hyperplane section of $G(2,5)$.
(c) X is a $\mathbf{P}^{(n+k)/2}$-bundle over a $(n-k)/2$-fold.

Throughout the paper, we shall assume that the base field K is algebraically closed and $\operatorname{char} K \neq 2$. We are using the results from §2 of [4]. Though we assume that the base field is the complex numbers in [4], those results and their proofs in §2 remain true under the condition that X is reflexive and $\operatorname{char} K \neq 2$.

Acknowledgement. I would like to thank Steve Kleiman for many helpful discussions and encouragements.

Received December 6, 1984. Research partially supported by a N.S.F. grant.