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ON THE SUPERCUSPIDAL REPRESENTATIONS OF GL_4, I

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The purpose of this paper, and of one which will follow it, is to show that every irreducible supercuspidal representation of the group $G = GL_4(F)$, F a p-adic field, may be constructed by induction from one of the three (up to conjugacy) maximal compact-modulo-center subgroups of G. We remark that the method of construction of supercuspidal representations by induction from open subgroups for the groups $GL_n(F)$ and related groups has a long history (see among others [M], [S], [MoS]) and that it is known that this method is successful in producing all irreducible supercuspidal representations for the groups $GL_n(F)$ [Mo] and $SL_n(F)$ [MoS] when $p \nmid n$ and for the groups $GL_l(F)$ [C] and $SL_l(F)$ [KS] when l is prime.

It is perhaps interesting to note that the problem of constructing all irreducible supercuspidal representations of $GL_n(F)$ by induction becomes more difficult when $p \mid n$ (the so-called "wild" case) or when n is composite (the composite case) but the problem becomes extremely difficult when both cases occur simultaneously. This may be explained as follows.

If $p \nmid n$, then one may, following Howe [H2], construct supercuspidal representations from so-called regular characters of anisotropic tori of $GL_n(F)$. The process is first to embed the multiplicative group $E^{\times}$ of an extension E over F of degree n into $GL_n(F)$ to obtain a torus, T_E say. There is then a unique maximal compact-modulo-center subgroup K containing T_E. Furthermore, given a regular character θ of $E^{\times}$ (and hence of T_E) there is an appropriate congruence subgroup K^r of K and a finite dimensional (often one-dimensional) representation κ of $T_E K^r$ which is naturally associated to θ such that $\mathrm{Ind}(GL_n(F), T_E K^r; \kappa)$ is irreducible supercuspidal. In case n is composite it is important to know to what extent the restriction of θ to the subgroups $1 + P_E^m$ of $E^{\times}$ factors through the norm from some field intermediate to E over F. This question is handled by Howe by his notion of *admissible character*.

On the other hand, if $p \mid n$, parameterization of irreducible supercuspidal representations by characters of tori fails for several reasons and it becomes necessary to analyze the decomposition of $\mathrm{Ind}(GL_n(F)K^r, \kappa)$ for certain representation κ of K^r, K^r as above. In particular, one may partition the set of (equivalence classes of) irreducible representations of K^r into three classes:

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