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EFFECTIVE CHABAUTY

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0. Introduction. By the Mordell–Weil rank of a curve over a number field we mean the rank of the group of points on its Jacobian. In his paper *Sur les points rationels des courbes algebriques de genre supérieur à l'unité*, Chabauty showed that a curve of genus g over a number field with Mordell–Weil rank at most $g - 1$ has finitely many points. This result is now superceded by Falting's work. However, as we shall show in this note, Chabauty's method can be used to give good effective bounds for the number of points on a curve as above. For example, we will show:

(i) A curve of genus two over $\mathbf{Q}$ with good reduction at two or three and Mordell–Weil rank at most one has at most twelve rational points.

More generally,

(ii) Suppose C is a curve of genus g over a number field K with Mordell–Weil rank at most $g - 1$. Suppose $\mathfrak{p}$ is an unramified prime of K at which C has good reduction, of residue characteristic strictly greater than $2g$. Then $\# C(K) \leqslant \mathbf{N}\mathfrak{p} + 2g(\sqrt{\mathbf{N}\mathfrak{p}} + 1) - 1$.

Turning the method around we can show:

(iii) Let k be an integer and $p > 2$ a prime not dividing k. Let $f(x)$ be a monic polynomial over $\mathbf{Z}$ such that $f(x) \equiv x^k \bmod p$ and $f(x)$ has at least $[(k + 1)/2]$ distinct roots in $\mathbf{Z}$. Then the Mordell–Weil rank of the curve

$$y^2 = f(x) + 1$$

is at least equal to its genus (which is $[(k - 1)/2]$).

I. Rational points and integrals of the first kind. Let p be a fixed rational prime and $\mathbf{C}_p$ the completion of a fixed algebraic closure of $\mathbf{Q}_p$.

Let K be a complete subfield of $\mathbf{C}_p$ and let A be an Abelian variety over K. As in [Co], §2, for each $\omega \in H^0(A, \Omega^1_{A/K})$ there exists a unique locally analytic homomorphism $\lambda_\omega : A(\mathbf{C}_p) \to \mathbf{C}_p$ such that $d\lambda_\omega = \omega$. If G is a subgroup of $A(K)$ let

$$V_G = \left\{ \omega \in H^0\left(A, \Omega^1_{A/K}\right) : \lambda_\omega(x) = 0 \ \forall x \in G \right\}.$$

Clearly V_G is a subspace of $H^0(A, \Omega^1_{A/K})$. If L is a finitely generated subgroup of

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