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# A CONDITION FOR MINIMAL INTERVAL EXCHANGE MAPS TO BE UNIQUELY ERGODIC

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**0. Introduction.** In this paper we consider a certain property (Property P) which an interval exchange map may satisfy (Definition 1.6). It turns out that minimal interval exchange maps having Property P must be uniquely ergodic (Theorem 1.7). Furthermore, given a family (a measure space) $\Omega$ of interval exchange maps (see Section 5), we define *Collective Property* P of $\Omega$ (Definition 5.1) which guarantees that "most" of the interval exchange maps in $\Omega$ have Property P (Theorem 5.2, which we call the Metric Theorem).

Our method (based on Theorems 1.7, 5.2) seems to be useful in ascertaining the unique ergodicity for "most" interval exchange maps depending on parameters and in the study of "rational billiards" (see [2] and [4], Sections 5 and 6). In the present paper we study interval exchange maps of rank 2 (Section 4), obtain a new proof of "Keane's conjecture" (Section 8), derive some metric results on the unique ergodicity of "most" finite group extensions of irrational rotations (see [13]), and establish a sufficient condition for a "rational" polygon $Q$ for "billiards" on $Q$ with one ball to be uniquely ergodic in Lebesgue almost all directions.

In Section 1 we set notation and state basic definitions and Theorem 1.7, the proof of which is given in Section 2. Then we show that the notion of Property P makes sense for arbitrary symbolic flows and that the analogue of Theorem 1.7 (Theorem 3.2) takes place in the context of symbolic flows.

In Section 4 we consider interval exchange maps of rank 2 (the lengths of all exchanged intervals are rational linear combinations of two real numbers, say $x$ and $y$). We prove that "typical" interval exchange maps of rank 2 must satisfy Property P (Theorem 4.5) and hence cannot be minimal but not uniquely ergodic (Theorem 4.4). (The restriction for a map to be "typical" is in fact a certain diophantine condition on $\alpha = x/y$. This condition is equivalent to the requirement for the $\alpha$-rotation of the unit circle to have Property P.) Note that the minimality implies the unique ergodicity for all interval exchange maps of rank 2 (even without restrictions on $\alpha$), however, the proof of this more general result (Theorem 4.1) is long and will be published elsewhere. The above results are applied for ascertaining the unique ergodicity of some finite group extensions of irrational rotations (see Theorem 4.7 which generalizes Theorem 1.2 in [13]). Then we consider "billiards" dynamical systems on the polygons whose sides are rational and whose angles are integral multiples of $\pi/2$. We observe that these

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