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CUSP FORMS ASSOCIATED TO LOXODROMIC ELEMENTS OF KLEINIAN GROUPS

IRWIN KRA

Let α and β be two distinct points in $\mathbf{C} \cup \{\infty\}$. Let

$$g_{\alpha,\beta}(z) = \frac{\alpha - \beta}{(z-\alpha)(z-\beta)}, \qquad z \in \mathbf{C} \cup \{\infty\}. \tag{0.1}$$

Observe that $g_{\alpha,\beta}$ is the unique (up to constant multiple) holomorphic automorphic form of weight (-2) for the one parameter group

$$\{A \in \mathrm{PSL}(2,\mathbf{C});\, A\alpha = \alpha, A\beta = \beta\}.$$

Let Γ be a finitely generated nonelementary Kleinian group with region of discontinuity Ω and limit set Λ. Fix an integer $q \geqslant 2$. Let $\mathbf{A}_q(\Omega,\Gamma)$ denote the space of cusp forms for Γ of weight $(-2q)$, the space of q-forms, for short.

Let $A \in \Gamma$ be loxodromic with attractive fixed point α and repulsive fixed point β. Let $\Gamma_0 = \langle A \rangle$ be the cyclic subgroup generated by A. Form the *relative* Poincaré series

$$\varphi_A = \theta_{\Gamma_0\backslash\Gamma}\, g^q_{\alpha,\beta};$$

that is,

$$\varphi_A(z) = \sum_{\gamma \in \Gamma_0\backslash\Gamma} g^q_{\alpha,\beta}(\gamma z)\gamma'(z)^q, \qquad z \in \Omega. \tag{0.2}$$

We call φ_A the *relative Poincaré series associated with the loxodromic element A* of Γ. It is easy to see that $\varphi_A \in \mathbf{A}_q(\Omega,\Gamma)$. This paper considers two closely related problems. Let $A_1, \ldots, A_N$ be N loxodromic elements of Γ and let $s_1, \ldots, s_N$ be complex numbers. Let

$$\varphi = \sum_{j=1}^{N} s_j \varphi_{A_j}. \tag{0.3}$$

(A) Find necessary and sufficient computable (algebraic) conditions for φ to be identically zero on Ω.

(B) Find necessary and sufficient conditions for $\varphi_{A_1}, \ldots, \varphi_{A_N}$ to be linearly independent.

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