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INVARIANTS OF MODULAR REPRESENTATIONS AND POLYNOMIAL ALGEBRAS OVER THE STEENROD ALGEBRA

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We are interested in the classification of unstable algebras over the mod p Steenrod algebra which are polynomial algebras. The classical examples are provided by the cohomology algebras of the classifying spaces of compact Lie groups (for large primes) but it is known that there are many examples which do not arise in this form. Moreover, not all unstable algebras over the Steenrod algebra can be realized as cohomology rings of spaces because there are strong restrictions on the behaviour of Steenrod powers on a cohomology ring due, for instance, to the existence of secondary and extraordinary operations. Hence, we would like to be able to display a list containing all irreducible unstable algebras over the mod p Steenrod algebra which are polynomial algebras, as well as a list of those algebras which can be realized as cohomology rings of spaces. This is indeed the situation in the *nonmodular* case, i.e., the case in which the generators of the polynomial algebra have degrees prime to p.

In the present paper we obtain some results on the classification of polynomial algebras as unstable algebras over the Steenrod algebra. In particular, we obtain a complete classification in the rank two case. The realizability problem is also considered. We obtain also an algebraic result of independent interest: we determine all two dimensional modular representations whose algebra of invariants is a polynomial algebra. Throughout this paper p will be a fixed odd prime. All algebras are over the field $\mathbf{F}_p$. $\mathscr{A}$ will denote the mod p Steenrod algebra. Since we are only concerned with polynomial algebras and p is odd we do not have to worry about the Bockstein. Hence, $\mathscr{A}$ might as well denote the quotient of the Steenrod algebra by the two sided ideal generated by the Bockstein, i.e., the subalgebra of the Steenrod algebra generated by the Steenrod powers.

1. Constructing unstable $\mathscr{A}$-algebras. In the rank one case it is easy to see that $\mathbf{F}_p[x]$ admits Steenrod operations if and only if $\deg x = 2p^i r$ and r divides $p - 1$. One can explicitly define the action of the Steenrod powers on $\mathbf{F}_p[x]$. However, as soon as we consider polynomial algebras with more than one generator, it is extremely difficult to construct unstable $\mathscr{A}$-actions by giving

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