Vol. 52, No. 2 DUKE MATHEMATICAL JOURNAL © June 1985

# ON EISENSTEIN SERIES OF HALF-INTEGRAL WEIGHT

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The definition of an Eisenstein series of $\mathrm{Sp}(m, \mathbf{Q})$ of half-integral weight is as follows. We first consider a theta series

$$\theta(z) = \sum_{x \in \mathbf{Z}^m} \exp(\pi i \cdot {}^t xzx),$$

where $z$ is the standard variable in the space $H_m$ of complex symmetric matrices with positive definite imaginary part. For $\gamma = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in \mathrm{Sp}(m, \mathbf{Q})$ with $a, b, c, d$ of size $m$, we write $a = a_\gamma$, $b = b_\gamma$, $c = c_\gamma$, $d = d_\gamma$, and define subgroups $P$ and $\Gamma_0(N)$ of $\mathrm{Sp}(m, \mathbf{Q})$ by

$$P = \{\gamma \in \mathrm{Sp}(m, \mathbf{Q}) \mid c_\gamma = 0\}.$$

$$\Gamma_0(N) = \{\gamma \in \mathrm{Sp}(m, \mathbf{Z}) \mid b_\gamma \equiv 0 \ (\mathrm{mod}\, 2),\ c_\gamma \equiv 0 \ (\mathrm{mod}\, N/2)\},$$

where $N$ is a positive integer divisible by 4. We can show that

$$\theta(\gamma(z)) = h_\gamma(z)\theta(z) \qquad \text{for every} \quad \gamma \in \Gamma_0(4)$$

with a factor of automorphy $h_\gamma$ such that $h_\gamma(z)^4 = \det(c_\gamma z + d_\gamma)^2$. Taking an odd integer $k$ and a Dirichlet character $\psi$ modulo $N$ such that $\psi(-1) = 1$, we consider a series

$$E(z, s; k/2, \psi, N) = \sum_{\gamma} \psi(\det(d_\gamma)) h_\gamma(z)^{-k} \det(\mathrm{Im}(\gamma(z)))^s,$$

where $z \in H_m$, $s \in \mathbf{C}$, and $\gamma$ runs over $[P \cap \Gamma_0(N)] \backslash \Gamma_0(N)$. We define also another type of series with respect to an arbitrary congruence subgroup $\Gamma$ of $\Gamma_0(4)$ by

$$E(z, s; k/2, \Gamma) = \sum_{\gamma \in (P \cap \Gamma) \backslash \Gamma} h_\gamma(z)^{-k} \det(\mathrm{Im}(\gamma(z)))^s.$$

In the present paper, we investigate the series of these types for $\mathrm{Sp}(m, F)$ with an arbitrary totally real algebraic number field $F$. Our main theorems in Section 2 will describe the behavior of $E$ at two critical points $s = 0$ and $s = (m + 1 - k)/2$ when $k > 0$. For simplicity, let us state here only the results in the easiest cases:

*Let $E(z, s)$ denote any one of the series of the above two types.*

Received August 20, 1984. Research supported by NSF Grant MCS 81-00744.