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LIFTING PROBLEMS AND LOCAL REFLEXIVITY FOR C^*-ALGEBRAS

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1. Introduction. Arveson was the first to discover the relevance of completely positive liftings to contravariant C^*-algebraic K-theory [4] (see also [5]). He showed that given a unital separable C^*-algebra A and a unital isomorphism φ of A into the Calkin algebra $Q = \mathscr{B}/\mathscr{K}$, φ has an inverse in the "Ext group" $\operatorname{Ext} A$ if and only if it has a completely positive unital lifting $\psi : A \to \mathscr{B}$. Subsequently it was shown that if A is nuclear, then all such maps lift, and thus for such A, $\operatorname{Ext} A$ is a group [12].

If A is not nuclear, J. Anderson proved that $\operatorname{Ext} A$ neet not be a group [1]. It had been shown previously that one could use an unusual tensor product obstruction to prove the nonexistence of completely positive liftings ([12], see also [25]). In this paper we will prove that this is the *only* obstruction to $\operatorname{Ext} A$ being a group. Letting $\otimes_{\min}$ denote the minimal (i.e., spatial) tensor product, we have:

THEOREM A. *For any separable unital C^*-algebra A, a unital extension*

$$0 \to \mathscr{K} \to B \to A \to 0$$

has an inverse in $\operatorname{Ext} A$ *if and only if for all C^*-algebras C, the corresponding sequence*

$$0 \to \mathscr{K} \otimes_{\min} C \to B \otimes_{\min} C \to A \otimes_{\min} C \to 0$$

is exact.

This result is a consequence of the following general lifting theorem (see Theorem 3.4). Throughout this paper, *ideals* are all assumed closed and two sided.

THEOREM B. *Suppose that J is a nuclear ideal in a unital C^*-algebra B. Then the following are equivalent:*

(1) *One can always lift a completely positive unital map $\phi : A \to B/J$ to a completely positive unital map $\psi : A \to B$ for any separable unital C^*-algebra A.*

(2) *The kernel of $B \otimes_{\min} C \to (B/J) \otimes_{\min} C$ is $J \otimes_{\min} C$ for all unital C^*-algebras C.*

We do not know if the nuclearity assumption on J is needed. Theorem B is still true if J is only approximately injective (see §3). As we show in §4, the latter is

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