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SOME NEW SURFACES WITH $P_g = 0$

REBECCA BARLOW

Introduction. The first example of a surface of general type with $p_g = 0$, $K^2 = 1$ was Godeaux's quotient $X = Q/Z_5$, where Q is a nonsingular quintic in P^3 on which Z_5 acts freely (1932).

Since the 1970s there has been interest in obtaining a complete classification of these surfaces. It is hoped that this kind of detailed treatment in a few cases will give insight into the layout of surfaces of general type with arbitrary values of p_g, q, and K^2.

An auxiliary invariant that has been used to distinguish between surfaces with the same numerical invariants is the torsion group $\operatorname{Tors} X = \operatorname{Tors} H_1(X, Z)$. Godeaux's surface X has Tors $= Z_5$, and Miyaoka showed that this is the largest possible torsion group, while Tors $= Z_2 \times Z_2$ does not occur.

Reid has now given an exhaustive construction for the cases Tors $= Z_3, Z_4, Z_5$ [R1]. In particular the moduli spaces for all three are shown to be irreducible and unirational, of dimension 8. Also $\pi_1 =$ Tors except possibly for Z_3.

The remaining cases Tors $= 1, Z_2$ were shown to exist in [Ba], [O-P] respectively. The moduli spaces for both are conjectured to be irreducible and are at least 8-dimensional by Kuranishi's theorem, but so far no complete families have been constructed. The example of Oort and Peters is a delicate "Campadelli double plane" construction, which does not appear to deform.

The main result of this paper is the construction in section 2 of a 4-parameter family of surfaces with $p_g = 0$, $K^2 = 1$, $\pi_1 = Z_2$. They are given as quotients Y/G, where Y is a complete intersection of 4 quadrics in P^6 and G is a group of order 16. The group G has a subgroup Z_8 acting freely, so that Y/G is double covered by the surface $T = Y/Z_8$ due to Godeaux, which has $p_g = 0$, $K^2 = 2$, $\pi_1 = Z_8$.

In sections 3 and 4 we give two similar constructions of 4-parameter families of surfaces with $\pi_1 = Z_4$, double covered by Godeaux–Reid surfaces with $p_g = 0$, $K^2 = 2$, $\pi = Z_8$, $Z_2 \times Z_4$ respectively. The existence of 4-parameter families of surfaces X with double covers T was conjectured by Reid: X has 4 nodes each of which should impose only 1 condition on the Kuranishi family.

The method used by Godeaux and then systematically by Reid in [R1] is to write down generators and relations for the unramified cover $Y \to X$ with Galois group $\operatorname{Tors} X$. This gets harder for smaller values of $\operatorname{Tors} X$, and has not yet succeeded for Z_2. In our constructions the Galois cover $Y \to X = Y/G$ is ramified, and the fundamental group of X is given by G/E where E is the elliptic subgroup (see §0). The key tool for obtaining the examples is the holomorphic Lefschetz fixed point formula. The resulting integrality conditions are also used

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