Vol. 51, No. 2 DUKE MATHEMATICAL JOURNAL © June 1984

FORMAL DECOMPOSITION OF n COMMUTING PARTIAL LINEAR DIFFERENCE OPERATORS

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§1. Introduction. Let $(e_1, \ldots, e_m)$ be a basis of the lattice $\mathbf{Z}^n$ in $\mathbf{C}^n$, and $z \in \mathbf{C}^n$. Consider the overdetermined system of partial linear difference equations:

$$u(z+e_i) - A_i(z)u(z) = 0, \qquad i = 1, \ldots, n. \tag{1}$$

Here the A_i are complex valued matrix functions of z, satisfying the relations:

$$A_i(z+e_j)A_i(z) = A_j(z+e_i)A_i(z). \tag{2}$$

These equations arise in the theory of Gauss–Manin connections: Let

$$\omega(z) = \sum_{j=1}^{n} \frac{z_j dP_j}{P_j}, \qquad z = (z_1, \ldots, z_m) \in \mathbf{C}^n, \qquad P_j \in \mathbf{C}[x_1, \ldots, x_k]$$

be a rational form on $\mathbf{C}^k$, S the zero set of $P_1 P_2 \ldots P_m$, then $\omega(z)$ defines for all $z \in \mathbf{C}^m$ a Gauss–Manin connection on $\mathbf{C}^k \setminus S$. Using the fact that under suitable hypotheses the hypercohomology of an associated DeRham complex vanishes, Aomoto shows in [1] and [2], that certain integrals of $P_1^{z_1} P_2^{z_2} \ldots P_n^{z_n}$ satisfy a system of type (1) with $A_i \in \mathrm{Gl}_n(\mathbf{C}(z))$.

He also solves partially the inverse problem: Recovering the integrals from the system (1). Note that if $A_i \in \mathrm{Gl}_m(\mathbf{C}(z))$ for all i, then there exists a meromorphic fundamental solution of (1) (see for instance Praagman [13]). However, in general it is difficult to find this solution explicitly. Therefore one proceeds in the following way: Solve (1) formally, and prove that there exists a unique solution, having this formal solution as asymptotic expansion as $z \to \infty$. In [1] Aomoto uses this technique, but has to allow serious restrictions on the A_i, in order to prove the existence of a formal solution. Precisely, the formal solution follows as a corollary of the following theorem:

THEOREM (1.1 *of* [1]). *Assume* $A_i \in \mathrm{Gl}_m(\mathbf{C}(z))$ *for all* i, *and for* $z_1 = \infty$, $z' = (z_2, \ldots, z_n)A_i$ *admits a Laurent expansion of the form*:

$$A_i(z) = A_{i0}(z')z_1^{\mu_i} + A_{il}(z')z_1^{\mu_i - 1} + \cdots$$

Received March 19, 1983. Revision received May 30, 1983. The author is supported by the Netherlands Foundation for Mathematics S.M.C. with financial aid from the Netherlands Organization for the Advancement of Pure Research.