Vol. 51, No. 2 DUKE MATHEMATICAL JOURNAL © June 1984

EXISTENCE OF ELASTIC DEFORMATIONS WITH PRESCRIBED PRINCIPAL STRAINS AND TRIPLY ORTHOGONAL SYSTEMS

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0. Introduction. There are two main results in this paper; one is a result in continuum mechanics and the other is in Riemannian geometry. The first result concerns the possibility of prescribing the principal strains of the finite deformation of an elastic medium. The second result is a proof of the existence of local orthogonal coordinates on three-dimensional Riemannian manifolds. Previously, both results had been proved in the analytic category (everything about the problems was required to be real analytic)—our contribution is to relax this requirement so that solutions are obtained to the problems in the C^∞ category.

The authors were motivated in at least three different ways in their work on these results. The first motivation was, of course, the intrinsic interest of the results themselves—the first problem is physically meaningful and the second result provides geometers and physicists with convenient coordinates in which to do computations on three-manifolds. The second motivation was to study systems of partial differential equations that interact in some way with the diffeomorphism group (see [D] for an example)—in the first problem we are searching for a diffeomorphism, and the second problem exhibits some curious invariance properties (see before (4.3)). A third motivation was to continue a program of extending results proved classically using the Cartan–Kähler theorem and known only in the analytic category to the C^∞-case (see [BGY] for the seminal example, also see [Y]).

The two problems that we discuss here are related by the analytic technique we use to solve them. It turns out that both can be reduced to nonlinear hyperbolic systems of partial differential equations with a particularly singular characteristic variety. In the three-dimensional version of both problems, the projectivized complex characteristic variety consist of three "real" (i.e., invariant under complex conjugation) lines in $\mathbb{C}P^2$ in general position. That this particular form should prevail in a stable way for nonlinear problems is, to say the least, surprising. The form of the problems allows us to apply the theory of symmetric hyperbolic systems [Fr] to their linearizations. Then, the Nash–Moser inverse function theorem can be applied to get the nonlinear theorems. The most concise

Received August 1, 1983. First author partially supported by NSF Grant #83-01095. Second author partially supported by an NSF Graduate Fellowship at Harvard University.