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A FATOU THEOREM FOR EIGENFUNCTIONS OF THE LAPLACE–BELTRAMI OPERATOR IN A SYMMETRIC SPACE

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1. Introduction. Assume $X = G/K$ is a Riemannian symmetric space of noncompact type. Here G is a semi-simple Lie group with finite center and K a maximal compact subgroup. With customary notations, as explained in Section 2, the generalized Poisson kernel of X is

$$P(gK; kM, H) = e^{-(\rho + H \mid H(g^{-1}k))}.$$

It is defined for $gK \in X$, $kM \in K/M$, the Furstenberg boundary of X, and $H \in \overline{\mathfrak{a}_+}$, the closure of the positive Weyl chamber.

With $H \in \mathfrak{a}_+$, we set for $\varphi \in L^1(K/M)$

$$P_H\varphi(gK) = \int P(gK; kM, H)\varphi(kM)\, dkM,$$

where dkM is the normalized K-invariant measure in K/M. Then $P_H\varphi$ is a joint eigenfunction of all G-invariant differential operators in X, with eigenvalues depending on H. If $H = \rho$, we get in particular strongly harmonic functions, that is, the eigenvalues are 0 for all invariant operators annihilating constants. One such operator is the Laplacian Δ of the Riemannian structure on X. For any H, one has

$$\Delta P_H\varphi = (\|H\|^2 - \|\rho\|^2)P_H\varphi$$

(see [3, Sec. 16–17]), where the Euclidean norm $\|\cdot\|$ comes from the Killing form in $\mathfrak{a}$. If we are interested in eigenfunctions of Δ only, it is thus enough to keep $\|H\| = R$ constant. We therefore fix $R > 0$, once and for all, and replace H by RH, with H varying over the set $S_+ = \{H \in \overline{\mathfrak{a}_+} : \|H\| = 1\}$. This gives us a large class of eigenfunctions of Δ. Indeed, Karpelevič [3, Theorem 17.2.1] has proved that any positive solution of the equation

$$\Delta u = (R^2 - \|\rho\|^2)u$$

in X is given as

$$u(gK) = P\mu(gK) \equiv \int P(gK; kM, RH)\, d\mu(kM, H)$$

for a unique positive measure μ in $K/M \times S_+$.

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