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ON THE ESSENTIAL SELFADJOINTNESS OF STOCHASTIC SCHRÖDINGER OPERATORS

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I. Introduction. There is a considerable mathematical interest in Schrödinger operators with random potentials in connections with the physics of disordered systems since the works of Pastur [1], [2], Gol'dshtein, Molchanov and Pastur [3], Kunz and Souillard [4] and others.

A problem of both mathematical and physical interest is the question of the essential selfadjointness of Hamiltonians of the type:

$$H_\omega = -\Delta + V_\omega \qquad \text{on } C_0^\infty(R^d)$$

where V_ω is a random field on R^d. This problem arose for example in our previous work [5] [6] [7] in connection with the problem of determining the spectrum of H_ω for a particular class of random potentials V_ω.

In the present paper we give a general criterion on V_ω in order that the corresponding Hamiltonian H_ω is essentially selfadjoint. More precisely H_ω is essentially selfadjoint on $C_0^\infty(R^d)$ almost surely provided there exists a constant A such that:

$$E\left\{\left[\left(\int_{C_y} |V_\omega(x)|^q \, dx\right)^{1/q}\right]^r\right\} \leqslant A < +\infty$$

for any unit cube C_y in R^d and suitable constants q and r, where E denotes the expectation with respect to ω. The proof of this result is given in section 2 while in the last section we discuss some examples arising in the theory of disordered systems.

After finishing this paper we became aware of Grenkova [12], where essential selfadjointness of random Schrödinger operators is proven under different conditions than ours.

II. The main theorem. Let $(\Omega, \mathscr{F}, P)$ be a probability space and let $V: \Omega \times R^d \to R$ be a jointly measurable function. We will write $V_\omega(x) = V(\omega, x)$. Denote by C_0 the cube:

$$C_0 = \left\{ x \in R^d;\ x = \sum_{j=1}^{d} t_j a_j,\ -\frac{1}{2} \leqslant t_j < \frac{1}{2} \right\} \tag{1}$$

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