Vol. 50, No. 4 DUKE MATHEMATICAL JOURNAL © December 1983

CALIBRATIONS ON R^6

JIRI DADOK AND REESE HARVEY

Table of Contents

1. Introduction. A form ϕ on n-dimensional euclidean space which is d-closed and has comass one is called a calibration. Associated with a calibration is a geometry of submanifolds consisting of those submanifolds on which ϕ is a volume form. Such ϕ-submanifolds are absolutely area minimizing. See Harvey–Lawson [1] for some interesting new ϕ-geometries. In order to understand the possible calibrated geometries, it is of basic importance to understand the parallel or constant coefficient calibrations. First, we recall the classical cases. If the degree is 1 or $n-1$, then the comass of ϕ is just the euclidean norm of ϕ. If the degree is 2 (or $n-2$), then ϕ (or $*\phi$) can be put in canonical form

$$\Phi \equiv \lambda_1 e_1 \wedge e_2 + \cdots + \lambda e_{2r-1} \wedge e_{2r}$$

with the comass equal to $\max_j |\lambda_j|$. Thus the simplest nonclassical case to consider is that of a 3-form ϕ in 6-variables. The object of this paper is to characterize all possible calibrated geometries arising from such a 3-form.

The exposed points of the mass ball $K \subset \Lambda^3 R^6$ are just the unit simple 3-vectors $G(3,6)$, and SO(6) acts transitively on these exposed points. Dually all of the maximal exposed faces of K^*, the comass ball in $\Lambda^3(R^6)^*$, are of the form $F^*(\xi)$ where $\xi \in G(3,6)$. Hence all the maximal exposed faces are the same under SO(6). Thus it suffices to describe $F^*(e_{123})$, the set of those forms of comass one attaining their maximum value one on e_{123}. First we note that the affine span of $F^*(e_{123})$ consists of those

$$\begin{aligned}\phi(A,\mu) = e_{123} &+ a_{11}e_{156} + a_{12}e_{416} + a_{13}e_{451}\\ &+ a_{21}e_{256} + a_{22}e_{426} + a_{23}e_{452}\\ &+ a_{31}e_{356} + a_{32}e_{436} + a_{33}e_{453}\\ &+ \mu e_{456}.\end{aligned}$$

Received May 4, 1983. Research supported by NSF Grants MCS-8101635 and MP575-05270.