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POINTWISE CONVERGENCE OF SINGULAR CONVOLUTION OPERATORS

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§1. Introduction. Let $K(z)$ be a classical Calderon–Zygmund kernel on $\mathbf{C}^n : K(z)$ is homogeneous of degree $-2n$, smooth away from 0, and of mean value zero on the unit sphere in $\mathbf{C}^n$. Form a distribution on the Heisenberg group, identified as usual with $\mathbf{C}^n_z \times \mathbf{R}_t$, by tensoring $K(z)$ with the Dirac measure $\delta(t)$. It has been shown by Geller and Stein [3] that the operation $f \to K(z)\delta(t) * f$ is bounded on $L^p(H_n)$ for $1 < p < \infty$, where $*$ denotes convolution with respect to the Heisenberg group structure. Furthermore, letting $K^\epsilon(z) = \chi\{|z| > \epsilon\}K(z)$, they showed that $K^\epsilon(z)\delta(t) * f \to K(z)\delta(t) * f$ in L^p as $\epsilon \to 0$. We show here that this convergence also occurs pointwise; the proof is far more complicated than the proof of pointwise convergence for ordinary Calderon–Zygmund operators. In Section 2 we recall the classical proof and apply it to the kernel $K(z)\delta(t)$, yielding a nontrivial "error" term which then becomes the focus of study. Using a strong maximal operator on H_n, this term is put in a simplified form, and in Section 3 the L^2 theorem is proved using the method of Kolmogorov–Seliverstoff and Plessner. The proof is geometrical in nature and seems likely to generalize to similar operators on the boundaries of strictly pseudoconvex domains in $\mathbf{C}^{n+1}$. The proof of the L^p theorem, presented in Section 4, makes use of the group Fourier transform and is less likely to generalize.

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§2. Preliminaries and method of proof. Let $H_n = \mathbf{C}^n_z \times \mathbf{R}_t$ with multiplication $(z,t) \cdot (z_1,t_1) = (z + z_1, t + t_1 - \frac{1}{2}\operatorname{Im} z \cdot \bar{z}_1)$ and convolution $f * g(z,t) = \int_{H_n} f((z_1, t_1))g((z_1,t_1)^{-1}(z,t))\,dz_1\,dt_1$. We sometimes write $(z,t) = (x,y,t)$ where $z = x + iy$, $x, y \in \mathbf{R}^n$. Let $K(z)$ be a Calderon–Zygmund kernel on $\mathbf{C}^n$, as in the introduction, and $K^\epsilon(z) = \chi\{|z| > \epsilon\}K(z)$ the standard truncations away from the origin. Let $K^\epsilon f = K^\epsilon(z)\delta(t) * f$, $Kf = K(z)\delta(t) * f$, defined initially for $f \in \mathscr{S}$.

THEOREM 1. *Let* $K^*f(z,t) = \sup_{0<\epsilon<\infty} |K^\epsilon f(z,t)|$. *Then*

$$\|K^*f\|_{L^2(H_n)} \leqslant c_K \|f\|_{L^2(H_n)}, \qquad f \in \mathscr{S}(H_n) \tag{1}$$

and thus $K^\epsilon f(z,t) \to Kf(z,t)$ *almost everywhere as* $\epsilon \to 0$ *for* $f \in L^2(H_n)$.

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