

ALGEBRAICALLY IRREDUCIBLE  
REPRESENTATIONS OF  $L^1$ -ALGEBRAS OF  
EXPONENTIAL LIE GROUPS

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**Introduction.** Let  $G$  be a locally compact group. The classical task of the representation theory of  $G$  is the determination of the strongly continuous irreducible unitary representations of  $G$ . These representations give, for example, in the unimodular type I case the ingredients in the Plancherel theorem. But for other purposes as the computation of spectra of  $L^1$ -functions considered as convolution operators on  $L^1(G)$  the knowledge of the irreducible unitary representations does not suffice. This is like in the case of an involutive commutative Banach algebra  $\mathcal{A}$  where in order to compute the spectrum of an element in  $\mathcal{A}$  one has to use the full Gelfand transformation, i.e., one has to deal with all multiplicative linear functionals and not only with the hermitian ones. It turns out that for the determination of spectra of functions in  $L^1(G)$  the proper class of representations to consider are the algebraically irreducible representations or, as I prefer to say, the simple  $L^1(G)$ -modules. Of course, in the case of commutative  $L^1$ -group algebras there is no difference between simple modules and irreducible unitary representations: every multiplicative linear functional is hermitian. But for noncommutative groups, even for solvable Lie groups, there are big differences. There exist (a lot of) solvable Lie groups  $G$  and simple  $L^1(G)$ -modules  $E$  such that the annihilator of  $E$  in  $L^1(G)$  is not the kernel of a unitary representation. The main result of this paper is a parametrization of the set of isomorphism classes of simple  $L^1(G)$ -modules for exponential Lie groups  $G$ . While simple modules play some role in the general theory of Banach algebras (for instance in the proof of Johnson's theorem on the uniqueness of the norm on a semisimple Banach algebra) the  $L^1$ -group algebras of exponential Lie groups are, up to my knowledge, the first example of a sufficiently wide class of noncommutative Banach algebras where all the simple modules can be determined explicitly. Of course, the simple modules were already known for  $L^1$ -group algebras of nilpotent Lie groups, but this case is less interesting in the sense that there is always a bijective correspondence between the simple modules and the irreducible unitary representations which means that one does not find new phenomena and a proper extension of the unitary representation theory. As a consequence of the parametrization theorem one obtains a characterization of the exponential Lie groups with symmetric  $L^1$ -group algebras.

This article is divided into six sections. In the first paragraph we summarize

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