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DIVISION ALGEBRAS, FIBRATIONS OF SPHERES BY GREAT SPHERES AND THE TOPOLOGICAL DETERMINATION OF SPACE BY THE GROSS BEHAVIOR OF ITS GEODESICS

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To Dock Sang Rim, who enriched our lives

Our main result is in Differential Geometry:

THEOREM A. *Any Blaschke manifold of dimension* $\leqslant 9$ *is homeomorphic to a sphere or projective space. In addition, any Blaschke manifold modelled on the Cayley projective plane* (*a* 16 *dimensional space*) *is homeomorphic to it.*

The method of proof translates the homeomorphism problem for Blaschke manifolds into a topological equivalence problem for smooth (of class C^{∞}) fibrations of spheres by great spheres. Our main results here are:

THEOREM B. *Any smooth fibration of* S^7 *by great* 3*-spheres or of* S^{15} *by great* 7*-spheres is topologically equivalent to a Hopf fibration.*

THEOREM C. *Any smooth fibration of* S^5 *or* S^7 *by great circles is topologically equivalent to a Hopf fibration.*

Theorem A follows from Theorems B and C. To prove Theorem B, we translate the fibration problem into one about division algebras, which has been solved by Buchanan. Theorem C is proved by a direct geometric construction and by an appeal to a theorem of Montgomery and Yang about free circle actions on the 7-sphere, or alternatively to Sullivan's classification theory for homotopy complex projective spaces.

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