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MIXED HODGE STRUCTURES ON PUNCTURED NEIGHBORHOODS

ALAN H. DURFEE

Introduction. Let Y be a complex projective algebraic variety and let $Z \subset Y$ be a subvariety. Let T be a topological neighborhood of Z in Y and let $T^* = T - Z$. The goal of this paper is to construct, in an elementary fashion, a mixed Hodge structure on the cohomology of T^*.

Special cases of this mixed Hodge structure have occurred previously. General results of Deligne show the existence of such a mixed Hodge structure when Z is an isolated singular point of Y. Clemens and Schmid put a mixed Hodge structure on certain punctured neighborhoods to prove the local invariant cycle theorem, and it had occurred to Schmid that it was possible to do this in general, although he had not worked out the details. Elzein, whose work has recently appeared, has obtained results in this area, and there is some overlap with this paper. Steenbrink has also obtained results on this type of mixed Hodge structure.

The purpose of Section 1 is to describe mixed Hodge complexes; these are chain complexes with filtrations whose cohomology groups have a mixed Hodge structure. We avoid derived categories and sheaves in order to keep the exposition elementary and in order to be able to construct mapping cones. The concept of a mixed Hodge complex for a topological space X is introduced; this is a mixed Hodge complex whose integral part is the singular chain complex of X. Also the standard mixed Hodge complexes for a smooth projective variety, a smooth variety, and a divisor with normal crossings are given, since these are the building blocks used later in the paper.

Section 2 is concerned with mapping cones, the central construction of the paper. Mapping cones are described in the category of chain complexes, filtered chain complexes, mixed Hodge complexes (where they yield a long exact sequence of mixed Hodge structures), topological spaces, and finally topological mixed Hodge complexes. The basic result in Proposition 2.7, which shows that a map of topological mixed Hodge complexes gives a long exact sequence of mixed Hodge structures whose integral part is the usual long exact cohomology sequence of a mapping cone. (Although the mapping cone is a special case of a simplicial object in Deligne's theory, it seems useful to have an elementary exposition of this topic.) The mixed Hodge structure on a variety with an isolated singularity is then derived in detail. Next, a similar result for the Mayer–Vietoris

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