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ON p-ADIC MEROMORPHIC FUNCTIONS

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§1. Introduction. 1.1. This paper arose out of attempts, in the first place, to construct a p-adic analog of Nevanlinna's theorem on the distribution of values of meromorphic functions, and, in the second place, to find a mechanism for analytic continuation of a meromorphic function which is given on a discrete sequence of points in the unit disc.

Classical Nevanlinna theory is so beautiful that one would naturally be interested in determining how such a theory would look in the p-adic case. There are two "fundamental theorems" which occupy a central place in Nevanlinna theory. In the present paper we shall only prove an analog of the first fundamental theorem. However, using results on p-adic interpolation, we shall obtain certain analogs of some important applications of the second fundamental theorem.

1.2. We now recall some facts from Nevanlinna theory. Let $f(z)$ be a meromorphic function in the complex plane C, and let $a \in C$ be a complex number. One asks the following question: How "large" is the set of points $z \in C$ at which $f(z)$ takes the value a or values "close to" a? For every a, Nevanlinna constructed the following functions. Let $n(f, a, r)$ denote the number of points $z \in C$ for which $f(z) = a$ and $|z| \leqslant r$, counting multiplicity. We set:

$$N(f,a,r) = \int_0^r \frac{n(f,a,t) - n(f,a,0)}{t}\, dt + n(f,a,0)\log r;$$

$$m(f,a,r) = \frac{1}{2\pi}\int_0^{2\pi} \log^+ \frac{1}{|f(e^{i\phi}) - a|}\, d\phi,$$

where

$$\log^+ x = \begin{cases} \log x, & \text{if } x > 1; \\ 0, & \text{if } x \leqslant 1. \end{cases}$$

We further set

$$T(f,a,r) = N(f,a,r) + m(f,a,r).$$

Nevanlinna's first fundamental theorem asserts that for every meromorphic

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