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A KINETIC CONSTRUCTION OF GLOBAL SOLUTIONS OF FIRST ORDER QUASILINEAR EQUATIONS

YOSHIKAZU GIGA AND TETSURO MIYAKAWA

Introduction. Consider the Cauchy problem for first order quasilinear equations of conservation type in the region $t \geqslant 0$:

$$u_t + \sum_{i=1}^{n} A^i(u)_{x_i} = 0, \qquad u(x,0) = u_0(x) \tag{M}$$

where $x = (x_1, \ldots, x_n)$, $u = u(x,t)$, and A^i, $i = 1, \ldots, n$, are real valued continuously differentiable functions of a single real variable. Usually the problem (M) is solved by using vanishing viscosity method or finite difference schemes. In this paper we give a new method to construct a global weak solution of (M). Our present work is inspired by Kaniel [7]. He tried to construct a global strong solution of the Cauchy problem for the Navier–Stokes equations, introducing a kinetic model for a monatomic gas. However, the details of his argument have not yet, to our knowledge, been fully checked.

To explain our method let us recall two concepts in gas dynamics: macroscopic v.s. microscopic. In the kinetic theory of gases one considers the Boltzmann equation which governs the change of the density distribution of gas particles in the phase space. The Boltzmann equation gives a microscopic description of thermofluid properties of gases, while conservation laws in fluid dynamics give a macroscopic one. Both descriptions are closely related. Integrating a microscopic quantity over its velocity argument yields an expression for a macroscopic quantity.

In this paper, instead of the Boltzmann equation, we consider a linear equation:

$$f_t + \sum_{i=1}^{n} \left(\psi^i(\xi) f\right)_{x_i} = 0, \qquad f(x,\xi,0) = f_0(x,\xi) \tag{m}$$

as a microscopic law, where $f = f(x,\xi,t)$, and $\psi^i(\xi)$ are functions of one real variable. A macroscopic quantity corresponding to f is defined by

$$v = \int_{-\infty}^{\infty} f(x,\xi,t)\, d\xi.$$

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