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THE BOUNDEDNESS OF THE MAXIMAL BOCHNER–RIESZ OPERATOR ON $L^4(\mathbf{R}^2)$

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1. Introduction. For $f \in \mathscr{S}(\mathbf{R}^2)$, let

$$\widehat{T_R^\alpha f}(\xi) = \left(1 - \frac{|\xi|^2}{R^2}\right)_+^\alpha \hat{f}(\xi)$$

where $\alpha \in \mathbf{R}$, $R \in \mathbf{R}^+$ and $\hat{}$ denotes the Fourier transform. Let

$$T_*^\alpha f(x) = \sup_{0<R<\infty} |T_R^\alpha f(x)|.$$

In this paper we prove the following:

THEOREM. *For each $\alpha > 0$, there exists a constant C_α such that*

$$\|T_*^\alpha f\|_4 \leqslant C_\alpha \|f\|_4 .$$

This theorem extends the Carleson–Sjölin theorem [1] and partially answers a question of E. M. Stein [7], p. 6. Using previously known results for L^2 and standard complex interpolation arguments, [8], we obtain $\|T_*^\alpha f\|_p \leqslant C_{p,\alpha}\|f\|_p$ if $\frac{1}{4} - \frac{1}{2}\alpha < (1/p) < \frac{1}{2} + \alpha$, $0 < \alpha < \frac{1}{2}$, and as a corollary, the almost everywhere convergence of the Bochner–Riesz means of order α for functions in L^p for the same range of p. These problems are open for $\frac{1}{2} + \alpha \leqslant (1/p) < \frac{3}{4} + \frac{1}{2}\alpha$, although for a lacunary sequence of R's, Córdoba and López-Melero [5] and Igari [6] have answered them affirmatively. During the final stages of preparation of this paper we have been informed that Córdoba has also obtained the above result.

The scheme of the proof is to reduce the problem to a Littlewood–Paley version of the Carleson–Sjölin–Fefferman–Hörmander multiplier theorem (see [3]), and then combine a geometric decomposition theorem for $\mathbf{R}^2$ with standard arguments to obtain the final result.

Throughout this paper, C and β will denote absolute constants, not necessarily the same at each occurrence.

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