GENERALIZATIONS OF DE FRANCHIS THEOREM

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0. Introduction. The purpose of this paper is to generalize the result due to de Franchis [2] which asserts that there are only a finite number of nonconstant holomorphic mappings of a fixed closed Riemann surface onto another fixed closed Riemann surface of genus > 1. The proof can be seen in Lang [11], Martens [12] and Samuel [16].

In §1, we shall prove that there are only a finite number of nonconstant holomorphic mappings of a fixed finite Riemann surface into another fixed finite Riemann surface of type (g,n) with $2g - 2 + n > 0$. (cf. Theorem 2.) The method is elementary and it owes to classical theorems of the function theory, that is, Fatou's Theorem and Riesz' Theorem for bounded holomorphic functions on the unit disc, and Tsuji's Theorem for Fuchsian groups of divergence type. (See Tsuji [21].)

By the same idea, we shall prove the following results in §2-6. Let X be a quasi-projective algebraic variety over C. Then we have:

(1) There are only a finite number of nonconstant holomorphic mappings of X into a fixed finite Riemann surface of type (g,n) with $2g - 2 + n > 0$. (cf. Corollary 2 of Theorem 3.)

(2) Let (M, π, S) be a holomorphic family of Riemann surfaces of type (g,n) with $2g - 2 + n > 0$ over a finite Riemann surface S of type (g_0, n_0) with $2g_0 - 2 + n_0 > 0$. Then the set of holomorphic mappings of rank 2 of X into M is finite. In particular, the analytic automorphism group of M is finite. (cf. Theorem 5 and its Corollary.)

(3) Let Γ be a fixed-point-free discrete subgroup of the analytic automorphism group of a strongly pseudoconvex domain Ω in C^n such that the quotient space Ω/Γ is compact. Then the set of nonconstant holomorphic mappings of X into Ω/Γ is finite. (cf. Corollary 2 of Theorem 6.)

(4) Let Γ be a fixed-point-free irreducible discrete subgroup of the n-th direct product $(PSL(2, R))^n$ of the real Möbius transformation group $PSL(2, R)$ acting on the upper half-plane U in the complex plane such that the quotient space U^n/Γ is compact. Then there are only a finite number of nonconstant holomorphic mappings of X into U^n/Γ. (cf. Theorem 7 and Noguchi–Sunada [14].)

(5) Let Γ be a fixed-point-free discrete subgroup of the analytic automorphism group of a bounded domain Ω in C^n. If nonconstant holomorphic mappings ϕ