Vol. 50, No. 1 DUKE MATHEMATICAL JOURNAL © March 1983

L^2-COHOMOLOGY AND INTERSECTION HOMOLOGY OF STRATIFIED SPACES

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Introduction. The main purpose of this paper is to clarify the relation which exists between the intersection homology of a singular space X with singularity Σ and the L^2-cohomology of $X - \Sigma$ with a suitable metric.

Specifically, let X be an n-dimensional compact stratified space with a fixed polyhedral structure and with a fixed stratification $X = X_n \supset X_{n-1} = X_{n-2} \supset X_{n-3} \supset \cdots \supset X_1 \supset X_0$, such that each stratum whose dimension is $j \leqslant n-2$ is diffeomorphic to the disjoint union of $(0,1)^j$'s. Let $\bar{p} = (p_2, p_3, \ldots, p_n)$, called a perversity, be a sequence of integers satisfying $p_2 = 0$ and $p_k \leqslant p_{k+1} \leqslant p_k + 1$, with $\bar{p} \leqslant \bar{m}$. Then what we want to assert is that the dual of the intersection homology group $IH_*^{\bar{p}}(X)$ with real coefficients defined by M. Goresky and R. MacPherson ([5], [6]) is canonically isomorphic to the L^2-cohomology group $H_{(2)}^*(X - \Sigma)$, where $X - \Sigma$ has a metric which is associated with $\bar{p}$ (Definition 4.4). The isomorphism is constructed through the medium of a kind of de Rham cohomology group of the complement of an open neighborhood of Σ in X, denoted $\mathcal{H}_{\bar{p}}^*(W_0(\epsilon))$ (Definition 4.7).

Here we make two remarks. First, the condition we imposed on each stratum of the stratification is necessary for describing $\mathcal{H}_{\bar{p}}^*(W_0(\epsilon))$ explicitly. Such an explicit description seems to be important because it gives concrete expression to the relationship between intersection homology and L^2-cohomology. Moreover, even if the strata do not satisfy the condition, by refining Σ, we can get the desired stratification. Second, the condition $\bar{p} \leqslant \bar{m}$ can be removed, but, in order to do so, we must generalize Lemma 3.12 to an assertion which includes the case $c < 0$.

We wish to point out that our result is a kind of generalization to singular spaces of the celebrated de Rham theorem on manifolds. Moreover, J. Cheeger showed in [3] that the L^2-cohomology group of a pseudomanifold X with triangulation T and with a piecewise flat metric is isomorphic to the dual of the intersection homology group of X with the middle perversity $\bar{m}$, namely, $H_{(2)}^*(X - \Sigma^{n-2}(T)) \cong (IH_*^{\bar{m}}(X))^*$, where $\Sigma^{n-2}(T)$ is the $(n-2)$-dimensional skeleton of T. Therefore our result is a generalization of his result.

In §1, §2 and §3, we explain stratified spaces, intersection homology and L^2-cohomology and study their basic properties. In §4 which is divided into five subsections, we study the relation between L^2-cohomology and intersection homology of stratified spaces. Our main results are Theorem 4.11, Theorem 4.13 and Theorem 4.14.

Received May 15, 1981. Revision received March 22, 1982