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THE LINDELÖF PRINCIPLE AND NORMAL FUNCTIONS OF SEVERAL COMPLEX VARIABLES

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Let $D \subset \mathbf{C}$ be the unit disc and $H^{\infty}(D)$ the customary space of bounded holomorphic functions. The classical Lindelöf principle is as follows [EH].

THEOREM 0.1. *Let $\gamma : [0, 1) \to D$ be a continuous curve satisfying $\lim_{t \to 1^-} \gamma(t) = P \in \partial D$. Let $f \in H^{\infty}(D)$ satisfy $\lim_{t \to 1^-} f(\gamma(t)) = l \in \mathbf{C}$. Then for any $\alpha > 1$ and Stolz region $\Gamma_{\alpha}(P) = \{z \in D : |z - P| < \alpha(1 - |z|)\}$ it holds that*

$$\lim_{\Gamma_{\alpha}(P) \ni z \to P} f(z) = l.$$

In other words, f has non-tangential limit l at P.

It may be noted that, in the classical literature, restrictions are sometimes imposed on γ (for instance, that it be Jordan). The following remarkable result of G. Whyburn [GW] obviates the need for this restriction.

PROPOSITION 0.2. *If $\gamma : [0, 1] \to \mathbf{R}^2$ is a continuous curve with $A = \gamma(0)$, $B = \gamma(1)$ then there is a sense-preserving, Jordan subcurve $\tilde{\gamma}$ of γ with $\tilde{\gamma}(0) = A$, $\tilde{\gamma}(1) = B$.*

In [WG], Gross used the elliptic modular function to extend Theorem 0.1 to meromorphic functions which omit three values. In [LV], Lehto and Virtanen established a much broader generalization. These authors were able to enlarge both the class of functions and the class of curves under consideration. What is perhaps more significant is that [LV] connects the Lindelöf principle with hyperbolic geometry. This fact will be important in the present paper.

The family of functions introduced in [LV] is called the *normal functions*. A meromorphic function $f : D \to \mathbf{C} \cup \{\infty\} \equiv \hat{\mathbf{C}}$ is said to be normal if $\{f \circ \varphi_{\alpha}\}_{\alpha \in A}$ is a normal family whenever $\{\varphi_{\alpha}\}_{\alpha \in A}$ is a set of conformal self-maps of the disc. It is proved in [LV] that Theorem 0.1 holds for all normal functions. These include, for instance, H^{∞} functions, schlicht functions, and meromorphic functions which omit three values.

The purpose of the present paper is to investigate normal functions, and various Lindelöf phenomena, on smoothly bounded domains in $\mathbf{C}^n$ when $n > 1$.

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