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APPROXIMATION OF SOLUTIONS OF LINEAR PDE WITH ANALYTIC COEFFICIENTS

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Introduction. The main result of this paper is the approximation formula (1.4). This result concerns the distribution solutions of a first-order system of homogeneous linear PDE with analytic coefficients, whose complex characteristics are simple. It implies that any such solution is the limit, in the appropriate distribution sense, of a sequence of *analytic* solutions of the same equations. Actually it is somewhat more precise, in so far as it expresses the analytic approximators in terms of the Cauchy data of the original solution. The result represents a generalization (in a certain direction) of the approximation formula in [Baouendi–Treves 1] (see also [Treves 3]).

We have limited ourselves to first-order determined systems. It will be obvious to the reader that we could as well have dealt with higher-order scalar differential operators or, for that matter, with higher-order determined systems. All that is required is a reduction, in all cases, to first-order determined systems with simple characteristics—but in general the latter equations will have to be pseudo-differential, which at first sight seems to preclude the use of the Cauchy–Kovalevska theorem (we use it repeatedly). But this difficulty can be easily circumvented—by returning to the original differential system each time—one needs to apply the Cauchy–Kovalevska theorem.

Also we have limited ourselves to proving distribution approximation. But a closer inspection of the reasoning will show that the limit in the approximation formula can be interpreted in a more restrictive sense if the regularity requirements on the solution are strengthened.

The proof of the approximation formula (1.4) relies on the techniques of analytic pseudodifferential and Fourier integral operators (with complex phases). We have gone to some length to explain the conceptual guidelines of the proof rather than presenting all the details of the manipulations of those operators, manipulations which are by now fairly standard (solutions of eikonal and transport equations, use of cut-off functions, etc.). Although we feel the proof is "natural" from beginning to end, it could not have been carried out without microlocalization.

Nor could it have been carried out without the hypothesis that the complex characteristics are simple. In its absence we would be hard pressed even to make a guess as to the validity of the approximation result. Clearly there are other cases where a formula like (1.4) is true, as for instance when the coefficients are

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