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CONTINUOUS LINEAR DIVISION AND EXTENSION OF $\mathscr{C}^\infty$ FUNCTIONS

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0. Introduction. There are two main aspects of this paper: division theorems generalizing Malgrange's results on ideals generated by analytic functions; and the existence of continuous linear solutions to several problems, including the division problems and problems of extending $\mathscr{C}^\infty$ functions defined on closed subanalytic sets.

Let Ω denote a real analytic manifold. (All manifolds are assumed countable at infinity.) Let $\mathscr{C}^\infty(\Omega)$ (respectively $\mathscr{O}(\Omega)$) denote the algebra of $\mathscr{C}^\infty$ (respectively real analytic) functions on Ω. Let $Y \subset X$ be closed subsets of Ω. Denote by $\mathscr{C}^\infty(\Omega, X)$ (respectively $\mathscr{E}(\Omega, X)$) the ideal in $\mathscr{C}^\infty(\Omega)$ of functions which vanish on X (respectively, which vanish on X together with all their partial derivatives). Put $\mathscr{C}^\infty(X,Y) = \mathscr{C}^\infty(\Omega, Y)/\mathscr{C}^\infty(\Omega, X)$ and $\mathscr{E}(X,Y) = \mathscr{E}(\Omega, Y)/\mathscr{E}(\Omega, X)$. Write $\mathscr{C}^\infty(X) = \mathscr{C}^\infty(X, \emptyset)$ and $\mathscr{E}(X) = \mathscr{E}(X, \emptyset)$. Both $\mathscr{C}^\infty(X,Y)$ and $\mathscr{E}(X,Y)$ are quotients of closed subspaces of the Fréchet space $C^\infty(\Omega) = \mathscr{E}(\Omega)$; hence they have natural Fréchet space structures.

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