

CONTINUOUS LINEAR DIVISION AND EXTENSION OF \mathcal{C}^∞ FUNCTIONS

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0. Introduction. There are two main aspects of this paper: division theorems generalizing Malgrange's results on ideals generated by analytic functions; and the existence of continuous linear solutions to several problems, including the division problems and problems of extending \mathcal{C}^∞ functions defined on closed subanalytic sets.

Let Ω denote a real analytic manifold. (All manifolds are assumed countable at infinity.) Let $\mathcal{C}^\infty(\Omega)$ (respectively $\mathcal{A}(\Omega)$) denote the algebra of \mathcal{C}^∞ (respectively real analytic) functions on Ω . Let $Y \subset X$ be closed subsets of Ω . Denote by $\mathcal{C}^\infty(\Omega, X)$ (respectively $\mathcal{A}(\Omega, X)$) the ideal in $\mathcal{C}^\infty(\Omega)$ of functions which vanish on X (respectively, which vanish on X together with all their partial derivatives). Put $\mathcal{C}^\infty(X, Y) = \mathcal{C}^\infty(\Omega, Y) / \mathcal{C}^\infty(\Omega, X)$ and $\mathcal{A}(X, Y) = \mathcal{A}(\Omega, Y) / \mathcal{A}(\Omega, X)$. Write $\mathcal{C}^\infty(X) = \mathcal{C}^\infty(X, \emptyset)$ and $\mathcal{A}(X) = \mathcal{A}(X, \emptyset)$. Both $\mathcal{C}^\infty(X, Y)$ and $\mathcal{A}(X, Y)$ are quotients of closed subspaces of the Fréchet space $C^\infty(\Omega) = \mathcal{C}^\infty(\Omega)$; hence they have natural Fréchet space structures.

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