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A LOCALLY FREE KLEINIAN GROUP

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The purpose of this note is to exhibit a Kleinian group which is locally free but not free. Specifically, we will find a discrete group of Möbius transformations G with the following properties.

1. G acts discontinuously at some point of the extended complex plane $\hat{\mathbf{C}}$.
2. G is locally free.
3. G is perfect (i.e. $G = [G, G]$, its commutator subgroup).
4. There is an invariant component Δ of the set of discontinuity Ω of G.
5. Δ / G is (conformally equivalent to) a plane domain.
6. G contains no parabolic elements.

Every group of Möbius transformations operates on hyperbolic 3-space H. Our construction gives us a hyperbolic 3-manifold $M = H/G$ with the following properties.

1′. M is topologically equivalent to the interior of a (non-compact) 3-manifold $\overline{M}$ with boundary.

2′. $\pi_1(M)$ is locally free.

3′. $H_1(M) = 0$.

4′. There is a component $S \subset \partial M$ so that the inclusion $i\colon S \to \overline{M}$ induces a surjection $i_* : \pi_1(S) \to \pi_1(\overline{M})$.

5′. S is topologically equivalent to a plane domain.

6′. Every free homotopy class of loops in M contains a shortest element.

We remark that it is not clear under which conditions the invariant component is all of Ω; i.e., $S = \partial M$.

§1. Construction of a plane domain. The easiest way to think of this plane domain is as follows. Start with a finite cylinder D^1. Attach a 3-holed sphere to each boundary; this yields a 4-holed sphere D^2. Attach a 3-holed sphere to each of these four holes; this yields an 8-holed sphere D^3. We continue in this manner to obtain $D = \cup D^n$, where $D^{n+1} \supset D^n$, D^n has 2^n boundary curves, and $D^{n+1} - D^n$ is the disjoint union of 2^{n-1} 3-holed spheres.

One easily sees that D is topologically equivalent to the complement of a Cantor set.

We choose a complex structure on D so that D is represented by a Fuchsian group of the first kind (see Keen [1] for the construction of such a group). In

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