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TOPOLOGICAL TRIVIALITY OF VARIOUS ANALYTIC FAMILIES

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A set-up which occurs often in algebraic geometry is the following: W^{n+1} is a complex analytic variety; D^* is the punctured disk in the complex plane; and $p: W^{n+1} \to D^*$ is a proper map whose differential is everywhere of rank 1. It follows that the fibers, $p^{-1}(t)$, are all compact analytic varieties which depend analytically on t. In addition, they form a differentiably locally trivial fiber bundle over D^*. Of course, one is usually interested in the case when the fibers are algebraic (or possibly projective) varieties.

All the differentiable information in the set-up is contained in the real submanifold sitting above a circle with center at the puncture. Restricting to this submanifold yields a differentiable fiber bundle

$$p: X^{2n+1} \to S^1.$$

Let $\partial/\partial\theta$ be the standard vector field on S^1. Well-known arguments using C^∞-partitions of unity allow us cover $\partial/\partial\theta$ by a vector field τ on X. Let $V^{2n} \subset X^{2n+1}$ be the fiber over the point of S^1 of argument 0. Integrating τ defines a map $\Phi: V \times [0,2\pi] \to X^{2n+1}$ so that

(a) $p \circ \Phi(v,t) = e^{it}$, $0 \leqslant t \leqslant 2\pi$

(b) $\Phi: V \times \{t\} \to p^{-1}(e^{it})$ is a diffeomorphism, and

(c) $\Phi: V \times \{0\} \to V$ is the identity.

Let $\varphi: V \to V$ be the diffeomorphism given by the following composition:

$$V = V \times \{2\pi\} \xrightarrow{\Phi} p^{-1}(2\pi) = p^{-1}(0) = V.$$

It is clear that if in $V \times [0,2\pi]$ we identify $(v,2\pi)$ with $(\varphi(v),0)$, then the quotient manifold is mapped diffeomorphically by Φ onto X:

$$\begin{array}{ccc} \Phi: (V \times [0,2\pi])/\{(v,2\pi) \sim (v,0)\} & \cong & X \\ \Big\downarrow \text{projection} & & \Big\downarrow p \\ [0,2\pi]/\{0 \sim 2\pi\} & = & S^1 \end{array}$$

The diffeomorphism $\varphi: V \to V$ is called the *monodromy of the family*. The map φ depends on the choice of vector field τ covering $\partial/\partial\theta$. As we vary τ we vary φ by

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