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A DEGENERATING FAMILY OF QUINTIC SURFACES WITH TRIVIAL MONODROMY

ROBERT FRIEDMAN

§0. Introduction. The purpose of this paper is to construct a family of quintic surfaces over the punctured disk, no base change of which may be filled in smoothly, and with trivial monodromy in homology. The family fills in complex-analytically with a singular fiber which is, essentially, a minimal surface of general type with $c_1^2 = p_g = 4$ meeting a $\mathbf{P}^2$ along a double curve which is a conic in the $\mathbf{P}^2$. The algebraic topology of the monodromy has been investigated by John Morgan, who shows in [7] that some power is homotopic (and even pseudoisotopic) to the identity. Using the results of §2 and the arguments in [7], there is a degeneration of algebraic varieties with general fiber a product of two quintic surfaces which cannot be filled in smoothly (for any base change), yet such that the monodromy is isotopic to the identity. In other words, the family over the punctured disk is a C^∞ product and the family fills in complex-analytically with a singular fiber, but the complex structure fails to fill in in an essential way.

To put this result in perspective, note that it is easy to construct degenerations of curves of genus > 1 with trivial monodromy on homology and such that no base change can be filled in smoothly. Essentially, all such are obtained by considering degenerations to reducible curves such that the dual graph is contractible. The general fiber is not simply connected, and it is easy to show that the monodromy has infinite order in homotopy. For higher dimensions, it is easy to construct degenerations of complex tori over the punctured disk which are C^∞-products and cannot be filled in smoothly. However, the general fiber is not simply connected and the families so constructed tend not to fill in complex analytically at all. The point of our example is that, in many ways, the general fiber is as pleasant as possible: it is simply connected, has very ample canonical bundle, and satisfies local Torelli. A consequence of the pathology exhibited here is that, in order to make the period map for quintics proper, it is essential to consider singular varieties.

In §1, the example is constructed, based on Horikawa's classification of numerical quintics and surfaces with small c_1^2. In §2 we prove that the example and variants of it cannot be filled in smoothly. §3 puts the example of §1 in a more general deformation-theoretic context, and §4 lists a few related problems.

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