Vol. 50, No. 1 DUKE MATHEMATICAL JOURNAL © March 1983

# A BEZOUT TYPE THEOREM FOR POINTS OF FINITE TYPE ON REAL HYPERSURFACES

JOHN P. D'ANGELO

**Introduction.** The purpose of this paper is to prove the first Bezout type theorem for points of finite type on algebraic real hypersurfaces of $\mathbf{C}^n$. Given an algebraic real hypersurface $M$, and a point $p$ in it, one can conclude that either there is a complex analytic variety $V$ containing $p$ and lying in $M$, or every such variety has finite order of contact, with a sharp bound depending on the degree of the hypersurface, and the ambient dimension.

There are two main tools used here. The first is the method of the author for estimating orders of contact in terms of multiplicities. The second is a result of Fulton, called the refined Bezout theorem, which can be used to estimate multiplicities. The author would like to thank Joe Harris for providing a reference to Fulton's result.

**Statement and Proof of the Theorem.** We recall the preliminary definitions and notations needed to discuss orders of contact of complex analytic varieties with a real hypersurface. Let $\mathcal{O}_p$ and $C_p^\infty$ denote the rings of germs of holomorphic functions at $p$ and the ring of germs of smooth real functions at $p$. If $M$ is a real hypersurface of $\mathbf{C}^n$, and $p$ lies in $M$, then the ideal in $C_p^\infty$ of functions that vanish on $M$ will be denoted by $(r_p)$. Of course, at each point $p$, this ideal is generated by the germ at $p$ of some local defining function $r$ for $M$. Note that $dr(p)$ does not vanish.

To define orders of contact, we pull back ideals to complex analytic curves, and measure orders of vanishing. Hence, write $\nu(g)$ for the order of vanishing of the function $g = g(p)$ at a point $p$. If $g$ happens to be vector valued, write $\nu(g)$ for the minimum order of the components. Let $\mathcal{C}_p^*$ denote the set of germs of non constant holomorphic maps $z : (\mathbf{C}, 0) \to (\mathbf{C}^n, p)$. One can think of elements of $\mathcal{C}_p^*$ as parameterized holomorphic curves passing through $p$. We remind the reader that $\nu(z)$ is a positive integer that is independent of the choice of coordinates and parameterization. It is of fundamental importance to allow $\nu(z)$ to be larger than one. We now define orders of contact.

1. *Definition.* Let $I_p$ be an ideal in either $C_p^\infty$ or $\mathcal{O}_p$. Let $z$ be an element of $\mathcal{C}_p^*$. We define the order of contact of the image of $z$ with $I_p$ by

$$\Delta(I_p, z) = \inf_{g \in I_p} (\nu(z^*g)/\nu(z)).$$

Received November 8, 1982. Partially supported by the NSF.