Vol. 50, No. 1 DUKE MATHEMATICAL JOURNAL © March 1983

REPRESENTATIONS OF GL(n) AND DIVISION ALGEBRAS OVER A p-ADIC FIELD

JONATHAN D. ROGAWSKI

Introduction. Let G be a reductive group over a local or global field F and let $\prod(G)$ denote the set of irreducible admissible (resp. automorphic) representations of G if F is local (resp. global). Let G' be an inner form of G defined over F. The conjectures of Langlands, embodied in his principle of functoriality with respect to the L-group, include as a special case the existence of a correspondence between $\prod(G)$ and $\prod(G')$. A precise conjecture is difficult to state, but in the case treated in this paper, that of a p-adic field F of characteristic zero, $G = \mathrm{GL}(n)$, and $G' = D^{\times}$ where D is a division algebra of dimension n^2 and central over F, the general conjecture is not needed and we prove the existence of an injective map $\psi : \prod(G') \to \prod(G)$ uniquely defined in terms of a relation between the characters of π' and $\psi(\pi')$. This is the main result of the paper (Theorem 5.8). For an appropriate choice of measures on G' and G, the formal degrees of π' and $\psi(\pi')$ are equal.

There are four principal ingredients in the proof. The first, Proposition 1.1, is a simplified form of the Selberg trace formula due to Deligne and Kazhdan. Their idea was related to me by Langlands and that is what encouraged me to look for a proof of the main theorem. The second is Theorem 2.4, a result about the transfer of orbital integrals between G and G'. Its proof is based on Shalika's germ expansion theorem and the theorem of [19]. Vigneras [22] has proved a general theorem characterizing orbital integrals on p-adic reductive groups and her result together with [19] gives Theorem 2.4 as a special case. The proof in §2, however, is self-contained. In §5, Proposition 5.5, it is shown that the character of a supercuspidal representation of any p-adic reductive group is non-zero on all elliptic Cartan subgroups. Although the proof of Theorem 5.8 is global in the sense that the Selberg trace formula is the key tool, some local results on the representation theory of GL(n) are needed and in §4, these are deduced from results of Bernstein, Zelevinsky, Casselman, Jacquet, and Langlands. Finally, a

Received September 25, 1982.