Vol. 50, No. 1 DUKE MATHEMATICAL JOURNAL © March 1983

TANGENT CONES TO TWO-DIMENSIONAL AREA-MINIMIZING INTEGRAL CURRENTS ARE UNIQUE

BRIAN WHITE

Introduction. Every rectifiable k-cycle in $\mathbf{R}^n$ bounds an area minimizing surface (integral current), which is known to be regular almost everywhere [A]. Examples show that the surface may have one or more singularities, but very little is known about the structure of such a singularity other than the existence of tangent cones to the surface at the singularity. That is, if T is an area-minimizing integral current with $0 \notin$ support (∂T) and if $r_i \to 0^+$, then the sequence $\boldsymbol{\mu}(r_i^{-1})_\#(T \llcorner \mathbf{B}(0, r_i))$ (obtained by restricting T to the ball of radius r_i and dilating by r_i^{-1}) contains a subsequence which converges to an area minimizing cone $\mathbf{C}$. Perhaps the most basic open question about singularities in area minimizing surfaces is whether such a cone must be unique. Or is it possible for a different subsequence to converge to another cone $\mathbf{C}'$? That could happen only if the surface were to spin or oscillate between two or more cones as it approached 0; since such spinning would appear to be wasteful of area, it seems unlikely. However, uniqueness of tangent cone has been proved in only a few situations: for 1-dimensional stationary varifolds [AA1], for two-dimensional area minimizing currents mod 3 and soap-film-like varifolds in $\mathbf{R}^3$ [T1, T2], for area minimizing hypersurfaces mod 4 in $\mathbf{R}^n$ $(n \leqslant 7)$ [W], and for arbitrary minimal surfaces at isolated singularities, provided at least one of the tangent cones satisfies an additional hypothesis [AA2]. (Unfortunately the hypothesis does not hold in all cases of interest [B].)

In this paper we prove uniqueness of tangent cones for two dimensional integral currents in $\mathbf{R}^n$. Using an idea of Reifenberg [R], we reduce the problem to an "epiperimetric" inequality. The epiperimetric inequality is proved by constructing a comparison surface from the graph of a multiple-valued harmonic function, the area of which we estimate in terms of the Fourier series of its boundary values.

I would like to thank my thesis advisor, F. J. Almgren, Jr., for his help and encouragement, and W. K. Allard and Frank Morgan for pointing out several mistakes.

1. Preliminaries. In addition to the standard notation of geometric measure theory [F] we use the following.

Received May 3, 1982.