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SOME EXAMPLES OF AUTOMORPHIC FORMS ON Sp_4

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In memory of Takuro Shintani

§1. Introduction. The purpose of this paper is to construct certain automorphic forms on Sp_4, the symplectic group in four variables (of genus or rank 2). These forms exemplify some of the complications that can be expected in considering automorphic forms on higher rank groups. In particular they violate the "strong multiplicity one" condition in a rather thorough way. Our construction involves the correspondences or "liftings" of automorphic forms provided by the theory of θ-series (oscillator representation) between members of reductive dual pairs. We describe the general idea.

We will use the representation-theoretic formulation of the notion of automorphic form, rather than classical language. Let k be a number field. Let V be a finite dimensional vector space over k, and let $(\, , \,)$ be an inner product (a nondegenerate symmetric bilinear form) on V. Let $O(V)$ denote the group of linear isometries of $(\, , \,)$. Let W_1 denote another finite-dimensional vector space over k, and let $\langle \, , \, \rangle_1$ denote a symplectic (nondegenerate, antisymmetric, bilinear) form on W_1. Let $\mathrm{Sp}(W_1)$ be the group of linear isometries of the form $\langle \, , \, \rangle_1$. Set $W = V \otimes W_1$. The tensor product of the forms $(\, , \,)$ and $\langle \, , \, \rangle_1$ defines a symplectic form $\langle \, , \, \rangle$ on W. Let $\mathrm{Sp}(W)$ be the isometries of this form. The groups $O(V)$ and $Sp(W_1)$ act on W in the obvious way. Their action clearly preserves the form $\langle \, , \, \rangle$, and each group clearly commutes with the other. Hence $(O(V), \mathrm{Sp}(W_1))$ is a pair of mutually centralizing groups in $\mathrm{Sp}(W)$. In fact, each of the 2 groups is the full centralizer of the other in $\mathrm{Sp}(W)$, so that $(O(V), \mathrm{Sp}(W_1))$ forms a dual pair in $\mathrm{Sp}(W)$ in the sense of [H1].

Let A denote the adele ring of k. We regard V, $O(V)$, etc., as algebraic groups defined over k. Let V_A denote the adelic valued points of V, and $O(V_A)$ or $O(V)_A$ the A-valued points of $O(V)$, and so forth. From now on for clarity we indicate the k-valued points of V by V_k, of $O(V)$ by $O(V_k)$ or $O(V)_k$, and so forth. The ring A is the restricted direct product of the completions k_p at the various primes, finite or infinite, of k. Similarly the groups V_A or $O(V_A)$ are restricted direct products of the groups of k_p-rational points V_{k_p} or $O(V_{k_p})$. We will usually abbreviate $V_{k_p} = V_p$ and $O(V_{k_p}) = O(V_p) = O(V)_p$, and so forth.

The groups $O(V_p)$, etc., are locally compact. Let $\mathscr{R}(O(V_p))$ denote the set of equivalence classes of irreducible admissible representations of $O(V_p)$. (When p is infinite, we understand equivalence to mean infinitesimal equivalence of the space of K-finite vectors, for any maximal compact subgroup $K \subseteq O(V_p)$.) The

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