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THE CANONICAL RING OF A VARIETY OF GENERAL TYPE

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Introduction. Our object in this paper is to study the canonical ring

$$\bigoplus_{d=0}^{\infty} H^0(X, dK_X)$$

of a smooth n-fold X of general type, and in particular to learn in what degrees to expect generators and relations for this ring. In the case of curves, Noether's theorem [S] covers the question of generators and states that

(1) The canonical ring of a curve X is always generated in degrees $\leqslant 3$.
(2) If $g \geqslant 3$, the canonical ring of X is generated in degrees $\leqslant 2$.
(3) If X is nonhyperelliptic, the canonical ring of X is generated in degree 1.

The work of Enriques and Petri [S] takes care of the relations, stating:

(1) The relations in the canonical ring of a curve X are always generated in weights $\leqslant 4$.
(2) If $g \geqslant 4$, the relations in the canonical ring of X are generated in weights $\leqslant 3$.
(3) If X is not hyperelliptic, trigonal, or a plane quintic, the relations in the canonical ring of X are generated in degree 2.

There is a beautiful generalization of the work of Enriques–Petri by Arbarello and Sernesi [A–S]. They prove that for a smooth n-fold X of general type, under the assumptions

(1) the canonical ring of X is generated in degree 1;

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