Vol. 49, No. 4 DUKE MATHEMATICAL JOURNAL © December 1982

ALEXANDER POLYNOMIAL OF PLANE ALGEBRAIC CURVES AND CYCLIC MULTIPLE PLANES

A. LIBGOBER

0. Introduction. As part of the study of fundamental groups of the complement to plane algebraic curves, Zariski had undertaken an investigation of branched coverings of $\mathbf{P}^2$. Zariski had shown that simple homological invariants of those coverings provide nontrivial invariants of the complement to the branching set. For example, if the fundamental group of the complement of a curve is cyclic, then all coverings branched over it have vanishing first Betti number. The branched coverings of the plane were called multiple planes by Italian algebraic geometers and rather detailed information was obtained by them for coverings of small degree ([6], [2]). Zariski devoted to the study of cyclic multiple planes works [20] and [21]. The main result of the latter can be formulated as follows:

ZARISKI'S THEOREM. *Let C be an irreducible algebraic curve of degree n in $\mathbf{P}^2$ given in affine part by equation $f(x, y) = 0$. Let*

$$z^k = f(x, y) \tag{0.1}$$

be the system of cyclic coverings F_k, branched over C (and possibly over the line L in infinity). Assume that singularities of C are only nodes and cusps (i.e., locally given by the equation $u^2 + v^2 = 0$ or $u^2 + v^3 = 0$) and C is transversal to the line L. Then the first Betti number of the desingularisation $\tilde{F}_k$ of F_k vanishes unless both the degree n of the curve C and the degree k of the covering are divisible by 6.

In [21] Zariski also gives a condition for nonvanishing of the first Betti number in the case when k and n are divisible by 6 in terms of position of cusps, which enable him to give a condition under which the fundamental group of the complement will be nonabelian.

Since then cyclic branched coverings showed their usefulness in the study of the complements to knots and links (see [8] for a survey of this extensive subject). In particular, there was found a formula connecting the Alexander polynomial of knots and links with Betti numbers of cyclic coverings.

The purpose of this paper is to apply those methods from knot theory to the study of branched coverings of $\mathbf{P}^2$. We extend Zariski's theorem in two directions. Firstly we consider coverings branching over an irreducible curve C which might possess any sort of singularities. Secondly the position of the line in

Received December 1, 1980. Revision received June 14, 1982. Research supported by NSF Grant.