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RATIONAL CONJUGACY CLASSES IN REDUCTIVE GROUPS

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Introduction. Let F be a perfect field, $\bar{F}$ an algebraic closure of F, and Γ the Galois group of $\bar{F}/F$. Let G be a connected reductive group over F. When $G = GL_n$, we have the following two results.

(1) Two elements of $G(F)$ are conjugate in $G(F)$ if and only if they are conjugate in $G(\bar{F})$.

(2) A conjugacy class in $G(\bar{F})$ that is defined over F contains an element of $G(F)$.

These results are not valid for all G. In fact most groups other than GL_n do not satisfy (1) (except over fields, such as finite fields, that are very simple from the point of view of Galois cohomology). Langlands [2] has pointed out that the difference between conjugacy and $\bar{F}$-conjugacy has important consequences for harmonic analysis on groups over local and global fields. One purpose of this paper is to introduce a notion of conjugacy, *stable conjugacy*, that is intermediate in strength between conjugacy and $\bar{F}$-conjugacy, and that agrees with $\bar{F}$-conjugacy for elements x such that the centralizer of the semi-simple part of x is connected (this is always the case if the derived group of G is simply connected). It seems likely that for cases in which stable conjugacy and $\bar{F}$-conjugacy differ, stable conjugacy will be what is needed for harmonic analysis.

However, the main purpose of this paper is to investigate the extent to which (2) breaks down. We give complete results when $\mathrm{char}(F) = 0$ (when $\mathrm{char}(F) \neq 0$, we can still prove our results for semi-simple classes). We use the work of Steinberg [9]. In the first place he observes that a group which satisfies (2) must necessarily be quasi-split over F. Next he proves that if G is quasi-split, semi-simple and simply connected, then (2) holds for semi-simple conjugacy classes. The main result of this paper, Theorem 4.4, says that if $\mathrm{char}(F) = 0$, G is quasi-split, and the derived group of G is simply connected, then (2) holds for all conjugacy classes of G. One might think that for semi-simple classes, this general result should be a corollary of Steinberg's theorem. Unfortunately this does not seem to be the case, and we use instead a somewhat roundabout method (still based, however, on the work of Steinberg).

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