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HOLOMORPHIC EXTENSION OF CR FUNCTIONS

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1. Introduction. Suppose M is a real hypersurface in $\mathbf{C}^n$ defined by the equation $r(p) = 0$. At a point $p \in M$, the Levi form is (up to a scalar factor) the quadratic form

$$\mathfrak{L}_p(w) = -\sum_{j,k=1}^{n} \frac{\partial^2 r}{\partial p_j \partial \bar{p}_k}(p) w_j \bar{w}_k$$

on the space

$$H_p(M) = \left\{ w \in \mathbf{C}^n \mid \sum \frac{\partial r}{\partial p_j}(p) w_j = 0 \right\}$$

A theorem of Lewy [L_1] states that if $\mathfrak{L}_p$ has a positive eigenvalue then, locally, a continuous CR function (i.e., a continuous function on M which is annihilated by all tangential anti holomorphic vector fields) extends to a holomorphic function in the open set defined by $r(p) > 0$. If $\mathfrak{L}_p$ has eigenvalues of both signs then locally every continuous CR function extends to both sides of M, and consequently is the restriction of a holomorphic function. In particular, we have a regularity result: all continuous CR functions are smooth in this case.

There have been many attempts at generalizing these results to manifolds of higher codimension (see [BI], [G_1], [HT], [HW], [L_2], [N], and [W]). For example, in [HW] it was shown that if the excess dimension of the Levi algebra near $p \in M$ is maximal, then there is an open set Ω in $\mathbf{C}^n$ with M locally contained in the closure $\bar{\Omega}$, such that each CR function extends to a holomorphic function on Ω. What has been missing up to now is a quantitative description of the set Ω. It is our purpose here to provide such a description.

Suppose M is a smooth (i.e., C^∞) submanifold of $\mathbf{C}^n$ of real codimension d. Let J be the complex structure map on the tangent space $T_p\mathbf{C}^n$. If $p \in M$ we let $T_p(M)$ denote the real tangent space of M at p. The complex vector space

$$H_p(M) = T_p(M) \cap JT_p(M)$$

is the holomorphic tangent space of M at p. If the complex dimension of $H_p(M)$ is constant for $p \in M$ we say that M is a *CR submanifold*. If the complex dimension of $H_p(M)$ is the minimal possible (i.e., $m = n - d$), we say that M is a *generic CR submanifold.*

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