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STABLE AND LABILE BASE CHANGE FOR $U(2)$

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Let E/F be a quadratic extension of local or global fields of characteristic 0, and $\mathbb{A}_E, \mathbb{A}$ the rings of adèles of E, F in the latter case. Denote by a bar or $\tilde{\sigma}$ the nontrivial element of the galois group $\mathrm{Gal}(E/F)$, and let G be the quasi-split form of $GL(2)$ defined by the twisted galois action of $\mathrm{Gal}(\bar{F}/F)$ ($\bar{F}$ is an algebraic closure of F) given by $\tau(g) = \tilde{\tau}(g)$ if the restriction of $\tilde{\tau}$ (in $\mathrm{Gal}(\bar{F}/F)$) to E is trivial, and $\tau(g) = w\,{}^t\tilde{\tau}(g)^{-1}w^{-1}$ if $\tilde{\tau}$ restricts to $\tilde{\sigma}$ on E. Here $w = \left(\begin{smallmatrix} 0 & 1 \\ -1 & 0 \end{smallmatrix}\right)$, ${}^tg$ denotes the transpose of g, and $\tilde{\tau}$ acts by mapping the matrix $g = (g_{ij})$ $(1 \leqslant i, j \leqslant 2)$ in $GL(2, \bar{F})$ (g_{ij} in $\bar{F}$) to the matrix $\tilde{\tau}(g) = (\tilde{\tau}(g_{ij}))$. Then $G(E) = GL(2, E)$ and $G(F)$ is the subgroup of σ-invariant g in $G(E)$, where $\sigma(g) = w\,{}^t\bar{g}^{-1}w^{-1}$.

If "upstairs" and "downstairs" refer to objects defined over E and F, the purpose of this work is to lift (L-packets $\{\pi\}$ of) admissible (locally) and automorphic (globally) representations π downstairs, to such representations π^E upstairs. The image consists of σ-invariant π^E, those with ${}^\sigma\pi^E \simeq \pi^E$ where ${}^\sigma\pi^E(g) = \pi^E(\sigma(g))$. Only one-half of the σ-invariant π^E are obtained by this lifting, in contrast with the base change theory of $GL(n)$ [1,4], where all $\tilde{\sigma}$-invariant π^E are obtained. More precisely, *there are two distinct liftings λ and λ_1 which inject the set of one-dimensional or discrete series L-packets $\{\pi\}$ downstairs into the set of π^E; the images of λ and λ_1 are disjoint and their union exhausts the set of σ-invariant representations π^E upstairs which are one-dimensional, discrete series or for which ${}^\sigma\pi^E$ is equivalent, but not equal, to π^E (globally and locally).* The central character of a σ-invariant irreducible one-dimensional or discrete series local (or global) representation π^E is trival on $F^\times$ (or the group $\mathbb{A}^\times$ of idèles of F), not only on $NE^\times$ (or $E^\times N\mathbb{A}_E^\times$).

To explain these results by means of the Langlands functoriality principle denote by G' the group $\mathrm{Res}_{E/F}G$ obtained from G by restricting scalars from E to F (thus $G'(F) \simeq G(E)$), and recall that the L-groups of G and G' are

$${}^LG = GL(2, \mathbb{C}) \rtimes W_{E/F}, \qquad {}^LG' = (GL(2, \mathbb{C}) \times GL(2, \mathbb{C})) \rtimes W_{E/F};$$

the Weil group $W_{E/F}$ (of $(z, \tau), z$ in $E^\times$ or the idèle class group $E^\times \backslash \mathbb{A}_E^\times, \tau$ in $\mathrm{Gal}(E/F)$) acts through $\mathrm{Gal}(E/F)$ by

$$\sigma(g) = w\,{}^tg^{-1}w^{-1}, \qquad \sigma((g, g')) = (\sigma(g'), \sigma(g)) \qquad (g, g' \text{ in } GL(2, \mathbb{C})).$$

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