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MORDELL–WEIL GROUPS OF ELLIPTIC CURVES OVER C(t) WITH $p_g = 0$ OR 1

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Throughout this paper, E will denote an elliptic curve over a function field K in one variable over C. We will assume that the j-invariant of E is nonconstant. Our goal is to use the Néron model $f: X \to S$ of E/K to study the group of rational points $E(K)$. We are especially interested in the cases when X is either a rational surface or a $K3$ surface.

Letting p_g denote the geometric genus of X, we have

$$\operatorname{rank} E(K) \leqslant 2p_g + 8\chi(\mathcal{O}_X) \tag{0.1}$$

(see (1.1)). The first section of the paper shows how $E(K)_{\text{tor}}$ affects (0.1). For example, if $E(K)_{\text{tor}} \neq \{0\}$, then (0.1) can be improved to read

$$\operatorname{rank} E(K) \leqslant 2p_g + 4\chi(\mathcal{O}_X).$$

Our most surprising result is that

$$\operatorname{rank} E(K) \leqslant 2(g-1)$$

unless $E(K)_{\text{tor}}$ is especially simple (one of a list of 19 groups—see Corollary 1.3 and (1.9)).

In the second section, we assume $K = \mathrm{C}(t)$, and we examine the groups $E(\mathrm{C}(t))$ in detail when $p_g = 0$ or 1. In each case, the results of §1 give us a finite list of possible groups. For rational elliptic surfaces ($p_g = 0$), it is easy to see that all possible groups occur. The major result of this paper (Theorem 2.2) is that the same is true for elliptic $K3$ surfaces ($p_g = 1$). We construct the required examples using the surjectivity of the period map for $K3$ surfaces of degree 2 (proved by Horikawa in [3]). This shows that there are precisely 65 possibilities for $E(\mathrm{C}(t))$ when $p_g \leqslant 1$.

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§1. We preserve the notation of the introduction. The genus g of S is also the irregularity of X (see [2, Proposition 1.36]), so that $\chi(\mathcal{O}_X) = 1 - g + p_g$. The basic

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