Vol. 49, No. 3 DUKE MATHEMATICAL JOURNAL © September 1982

P-ADIC MEASURES FOR SPHERICAL REPRESENTATIONS OF REDUCTIVE P-ADIC GROUPS

MICHAEL HARRIS

Introduction. In recent years, much effort has been devoted to the construction of p-adic analogues of the L-functions attached to automorphic forms. In the work of Mazur–Swinnerton-Dyer [M-S] and Manin [Ma], among others, a crucial step is the translation of the relations imposed upon the various transforms of a given automorphic form by the action of the Hecke algebra into the "distribution relations" which characterize a p-adic measure. The articles just cited accomplished this translation for automorphic forms on GL(2); more recently, Masur [M] has succeeded in extending this technique to GL(n), for arbitrary n.

In attempting to understand Mazur's work, it is natural to reinterpret his construction in the language of reductive p-adic groups; it then becomes an exercise in the structure theory of such groups to perform this construction in the most general context. In this paper we carry out this exercise: we show that any "ordinary" spherical representation of a reductive p-adic group G (by "ordinary" we mean: definable over a p-adic integer ring) gives rise to a p-adic measure on the "integral points" of a maximal unipotent subgroup of G. The proof is based upon the known structure of the Hecke algebra of G with respect to a good maximal compact subgroup K; this theory is due essentially to Satake [Sa], and we have followed the exposition of MacDonald [Mac] rather closely.

The results in this paper thus represent the skeleton of a general theory of p-adic L-functions associated to automorphic forms. The skeleton will remain without flesh until this theory can be connected with that of complex L-functions, attached to automorphic forms. This has so far been carried out only for GL(2) and, in certain cases, for GL(3). The difficulties here are of several types: (1) In most cases one does not yet know how to define the complex L-function; therefore one cannot even make an intelligent guess as to what the p-adic analogue should be; (2) When the complex L-functon exists, its special values must be computable in terms of a generalized "modular symbol"; (3) The invariance properties of the symbol are often likely to make the measure equal zero; therefore the construction will have to be modified in each case to take this invariance into account (some possible modifications are described in §4, below); (4) The construction breaks down completely in the non-ordinary case, which indicates that it is not in its most convenient form (in Manin's paper [Ma], for

Received August 20, 1981. Partially supported by NSF Grant PCM77-04951.