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BOUNDARY REGULARITY OF PROPER HOLOMORPHIC MAPPINGS

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1. Introduction. A smooth bounded domain D contained in $\mathbb{C}^n$ is said to satisfy condition R if the Bergman projection associated to D maps $C^\infty(\bar{D})$ into $C^\infty(\bar{D})$. The purpose of this paper is to prove

THEOREM 1. *If $f: D_1 \to D_2$ is a proper holomorphic mapping between smooth bounded pseudoconvex domains contained in $\mathbb{C}^n$, and if D_1 satisfies condition R, then f extends smoothly to $\bar{D}_1$.*

Theorem 1 was proved in the special case that the mapping f is biholomorphic in [3]. Allowing the mapping to be proper creates obstacles which do not seem to be surmountable using the machinery of [3]. Hence, we are forced to develop new techniques to study boundary behavior of proper mappings. K. Diederich and J. E. Fornaess have announced that they have also obtained a proof of theorem 1.

Kohn's formula $P = I - \bar{\partial}^* N \bar{\partial}$ relates the Bergman projection P to the $\bar{\partial}$-Neumann operator N. Hence, whenever the $\bar{\partial}$-Neumann operator associated to a domain satisfies global regularity estimates, that domain satisfies condition R. J. J. Kohn has shown [16, 17, 18] that the $\bar{\partial}$-Neumann operator satisfies stronger estimates than global regularity estimates in a variety of cases. For example, the $\bar{\partial}$-Neumann operator associated to a smooth bounded domain D satisfies *subelliptic* estimates whenever D is strictly pseudoconvex [16], or D is pseudoconvex and of finite type in C^2 [17], or, more generally, whenever the boundary of D satisfies certain geometric conditions [18]. Diederich and Fornaess [9] have shown that these geometric conditions are satisfied by smooth bounded pseudoconvex domains with real analytic boundaries. Condition R can also be shown to hold for smooth bounded complete Reinhardt domains [6].

It is proved in [4] that if $f: D_1 \to D_2$ satisfies the hypotheses of theorem 1, then $u = \mathrm{Det}[f']$ extends smoothly to $\bar{D}_1$ and uf^α extends smoothly to $\bar{D}_1$ for each multi-index α. We shall take this as our starting point. Note that if u and uf^α extend *holomorphically* past bD_1 for each α, then the solution of a simple division problem in the ring of germs of holomorphic functions renders that f also extends holomorphically past bD_1. This procedure is described in detail in [5]. We intend to mimic this procedure in the $C^\infty(\bar{D}_1)$ category. Additional complications arise because the ring of germs of C^∞ functions does not form a unique factorization domain, as does the ring of germs of holomorphic functions. However, we shall be considering germs of C^∞ functions which are holomorphic on one side of a real hypersurface, and this ring does retain certain weak factorization properties.

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