Vol. 49, No. 2 DUKE MATHEMATICAL JOURNAL © June 1982

ON $J_1(p)$ AND THE CONJECTURE OF BIRCH AND SWINNERTON–DYER

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§1. Introduction. The conjecture of Birch and Swinnerton Dyer predicts that the L-series $L(A, \psi, s)$ of an abelian variety A vanishes at $s = 1$ if and only if the Mordell–Weil group of A, over the field cut out by ψ, is finite. In [9] Barry Mazur proves an analogue of the Birch and Swinnerton–Dyer conjecture, modulo an Eisenstein prime, for $J_0(p)$. Recently, Kamienny and Stevens [5] have extended a portion of Mazur's result to $J_1(p)$. They produce a congruence formula satisfied by the algebraic part $\Lambda_f(\psi)$ of the special values $L(f, \psi, 1)$ attached to weight-two new forms on $J_1(p)$. In this paper we prove that if the Eisenstein ideal is locally principal and $\Lambda_f(\psi) \not\equiv 0 \pmod{\mathfrak{q}}$, then the Eisenstein quotient associated to $\mathfrak{q}$ has a finite Mordell–Weil group over the field cut out by ψ when ψ is an odd quadratic character. We give two proofs of this fact. One is subject to certain numerical conditions, but yields the flat cohomology of the kernel of the Eisenstein prime. The other eliminates the numerical conditions but does not explicitly give us the flat cohomology groups.

Portions of sections 2, 3, and 4 appear in [4]. They are repeated here for the convenience of the reader. We should point out that the definition of the Eisenstein ideal I that we give here differs from the one given in [4]. There we studied the annihilator of the 0-cusps, while in this paper we consider I to be the annihilator of the ∞-cusps.

Some of the results of this paper were contained in the author's Harvard thesis [3]. He would again like to thank his advisors, Barry Mazur and Andrew Wiles, for their assistance and encouragement during the course of this work.

§2. Modular curves and Hecke operators. Let p be a prime number $\geqslant 13$. We recall the definition of two congruence subgroups of $SL_2(\mathbf{Z})$:

$$\Gamma_0(p) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL_2(\mathbf{Z}) : c \equiv 0 \pmod{p} \right\}$$

$$\Gamma_1(p) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma_0(p) : a \equiv d \equiv 1 \pmod{p} \right\}.$$

If $\mathfrak{H}$ is the upper half-plane we let $Y_1(p)_{\mathbf{C}}$ (respectively, $Y_0(p)_{\mathbf{C}}$ be the open

Received November 11, 1981.