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SEMIGROUPS AND BOUNDARY VALUE PROBLEMS

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Contents

Introduction. Let D be a bounded domain in $\mathbf{R}^N$ with smooth boundary ∂D and let $C(\bar{D})$ be the space of real-valued continuous functions on $\bar{D} = D \cup \partial D$.

A strongly continuous semigroup $\{T_t\}_{t \geq 0}$ of bounded linear operators on $C(\bar{D})$ is called a *Feller semigroup* on $\bar{D}$ if $\{T_t\}$ satisfies the following condition:

$$f \in C(\bar{D}), \qquad 0 \leq f \leq 1 \text{ on } \bar{D} \Rightarrow 0 \leq T_t f \leq 1 \text{ on } \bar{D}.$$

It is known (cf. [2], [5], [25]) that there corresponds to a Feller semigroup $\{T_t\}_{t \geq 0}$ on $\bar{D}$ a strong *Markov process* $\mathfrak{X}$ on $\bar{D}$ whose transition function $P(t,x,dy)$ satisfies

$$T_t f(x) = \int_{\bar{D}} P(t,x,dy) f(y), \qquad f \in C(\bar{D}) \tag{0.1}$$

and that, under certain continuity hypotheses concerning the transition function $P(t,x,dy)$ such as

$$\lim_{t \downarrow 0} \frac{1}{t} \int_{|y-x|>\epsilon} P(t,x,dy) = 0 \qquad \text{for all } \epsilon > 0 \text{ and } x \in \bar{D}, \tag{0.2}$$

the infinitesimal generator $\mathfrak{A}$ of $\{T_t\}_{t \geq 0}$ is described analytically as follows:

(i) Let x be a fixed point of the *interior* D of the domain. For a C^2-function u in the domain $\mathfrak{D}(\mathfrak{A})$ of $\mathfrak{A}$, by expanding $u(y) - u(x)$, we obtain from (0.1)

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