Vol. 49, No. 1 DUKE MATHEMATICAL JOURNAL © March 1982

TEMPERED REPRESENTATIONS AND ORBITS

W. ROSSMANN

Introduction. A *basic representation* π (induced from a limit-of-discrete-series) of a real reductive group G (which we shall here assume to be algebraic) corresponds to a family Λ of orbits in the imaginary dual $i\mathfrak{g}^*$ of the Lie algebra $\mathfrak{g}$ of G in a manner described in [ROS] (and explained below). The decomposition $\pi = \Sigma\pi_i$ of π into irreducibles π_i one would expect to correspond to a decomposition $\Lambda = \cup\Lambda_i$ of Λ into orbit packets Λ_i. We shall prove that this is indeed the case, at least when π is induced from discrete series, as we shall now assume: $\Lambda = \cup\Lambda_i$ is the decomposition of Λ into orbits of a certain subgroup P' of the *rho-group* P, which permutes Λ simply transitively. The quotient group P/P' therefore permutes the orbit packets Λ_i simply transitively; P/P' is isomorphic to the dual $\hat{R}$ of the Knapp-Stein R-group of π. As a by-product we also determine the decomposition $\pi = \Sigma\pi_i$, thus giving an alternative proof of a result of Knapp and Zuckerman ([K/Z1] Theorem 1). As pointed out there, the decomposition of these representations induced from discrete series provides (together with results of Harish-Chandra, Langlands, and Trombi) a classification of the tempered representations of G. As in the Knapp-Zuckerman approach, the main ingredients in the proof are a version of Schmid's character identity ([K/Z1], [SCH]; see section 2 below), and Harish-Chandra's bound for the intertwining number ([HC2], section 40). We shall also use Langlands's description of the R-group from [LA2], which in turn refers to Harish-Chandra's definitions in [HC2]. In addition we shall use some results of Knapp and Zuckerman on the structure of the R-group. On the other hand, we do not need the theory of Knapp and Knapp-Stein on intertwining operators [KNA], [K/S], (but intertwining operators enter in implicitly through [HC2]) nor do we need the theory of Knapp and Zuckerman [K/Z] (except for the auxiliary results mentioned above). Of course, there is no suggestion that our results on characters and orbits will take the place of deep facts on intertwining operators.

The paper is organized as follows. The first section contains the main definitions and results and recapitulates some essential background from [ROS][1]; the remainder contains the details of the proofs.

1. Definitions and results. Let G be the real subgroup of a complex, algebraic group $\mathbf{G}$ defined over R: $G = \{a \in G \mid \sigma_{\mathbf{G}}(a) = a\}$. Here $\sigma_{\mathbf{G}}$ is the complex

Received April 20, 1981

[1]The notation here has sometimes been changed from that of [ROS], so as to conform to the customary notation for algebraic groups.