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LIMIT ORBITS IN REDUCTIVE LIE ALGEBRAS

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Introduction. The questions considered here have their origin in the representation theory of reductive Lie groups. The relevant background can roughly be explained as follows.

An "admissible" tempered representation π of a reductive group G corresponds to a family L of principal orbits in the dual of the Lie algebras of G by means of a character formula, which expresses the character of π in exponential coordinates in terms of the invariant measures on the orbits in L [ROS1]. For a "basic" representation with regular parameters (see [K/Z]) L contains only one orbit; for a representation with singular parameters, however, L may contain several orbits, which are certain "limits" of regular ones; the decomposition of the representation into irreducible subrepresentations corresponds to a decomposition of L into certain subfamilies. To clarify the situation one is lead to the problem of classifying limit orbits.

There is another way leading from representations to limit orbits. The character of an irreducible admissible representation admits (in exponential coordinates) an asymptotic expansion in terms of distributions supported on certain limit orbits (see [B/V] and [K/V]). The nature of the orbits is related to properties of the representation (Gelfand–Kirillov dimension, asymptotic K-spectrum). Again one is lead to the problem of classifying limit orbits.

Even though limit orbits arose in representation theory, their classification is a purely geometric matter, which may be of interest in other connections. For this reason it seems best to separate the representation theory from the geometry altogether. As a result, representations are referred to only incidentally in this paper. Applications to representation theory will be given elsewhere ([ROS2]).

The paper is organized as follows. The first section summarizes the essential definitions and results; the remainder contains the details of the proofs. The proofs rely on results of Kostant on the orbit structure in a semisimple complex Lie algebra [KOS]; related results for real Lie algebra are due to Rothschild [ROT] (but will not be needed here; they are a byproduct of the classification of limit orbits). Some results of Matsumoto [MAT] are used in a less serious way.

Definitions and results. Of the real Lie group G we require that

(G1) the Lie algebra $\mathfrak{g}$ of G is reductive;
(G2) $\mathrm{Ad}(G)$ is contained in $\mathrm{Int}\,(\mathfrak{g})$.

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