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GROWTH OF TWISTED LAURENT EXTENSIONS

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0. Introduction. Let A be an algebra generated over the field k by the finite set S. Let A_n denote the k subspace of A spanned by all products of n or fewer elements of S. The growth function of A with respect to S is

$$\gamma(n) = \dim A_n.$$

Of course, $\gamma(n)$ depends on the choice of S, but its essential behavior is independent of this choice (cf. [13] for this and other basic facts about growth). In particular, the following definitions do not depend on S.

(1) A has *polynomially bounded* (*p.b.* for short) *growth* if there exists a polynomial $p(x)$ such that $\gamma(n) \leqslant p(n)$ for all n.

(2) A has *exponential growth* if there exists a constant a such that $\gamma(n) \geqslant a^n$ for all n sufficiently large.

Clearly, p.b. growth and exponential growth are mutually exclusive. Analogous concepts can be defined for groups. In fact, the growth of a group is exactly the growth of its group algebra. Gromov [3] has recently shown that the groups with p.b. growth are precisely the nilpotent-by-finite groups. It is unlikely that algebras with p.b. growth can be so succinctly characterized, but such characterizations may be possible if one restricts to suitable classes of algebras. The purpose of this note is to suggest such a characterization for twisted Laurent extensions of algebras with p.b. growth. Let σ be a k-algebra automorphism of the k algebra B. Let $A = B[u,u^{-1};\sigma]$ denote the algebra of all formal sums $\sum_{i=-m}^{n} u^i b_i$ where $b_i \in B$, with termwise addition and with multiplication determined by that in B together with the rule $bu = ub^\sigma$ for $b \in B$. A is called a twisted Laurent extension of B.

CONJECTURE. *Suppose B has p.b. growth. Then A has p.b. growth if and only if σ satisfies the following condition*:

(*) *There exists a sequence*

$$k = B_0 \subseteq B_1 \subseteq \cdots \subseteq B_n = B$$

of subalgebras of B such that for each $i = 1, 2, \ldots, n$, B_i is generated as an algebra by a finitely-generated left B_{i-1} module which is invariant under σ and σ^{-1}.

The remainder of this paper offers evidence supporting this conjecture. Of course, the evidence presented might also support weaker conjectures, such as that B sits "tightly" in an overring (e.g., a ring of quotients) to which σ can be extended so that (*) holds.

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