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ON THE INNER PRODUCT OF TRUNCATED EISENSTEIN SERIES

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Introduction. One can think of Eisenstein series as the spectral kernels for the Laplace–Beltrami operator on a certain class of noncompact Riemannian manifolds. They are the eigenfunctions corresponding to the continuous spectrum. In particular they are not square integrable. However, there is a natural way to truncate these functions so that they are square integrable. The object of this paper is to investigate the inner product of two such truncated functions. Our main result is an asymptotic formula for the inner product, as the variable of truncation approaches infinity. The formula, which is based on an inner product formula of Langlands for cuspidal Eisenstein series, is rather simple. It is given in terms of certain operators which are analogues of the classical scattering matrix.

The most efficient way to work with Eisenstein series is through adèle groups. The close connection between the analysis on adèle groups and that on locally symmetric Riemannian manifolds is well known and will not be discussed here. Let G be an algebraic group defined over Q, which for the introduction we take to be semisimple, and let $P = N_P M_P$ be a standard parabolic subgroup of G. If A_P is the split component of the center of the Levi component M_P, let $\mathcal{Q}_P$ be the space of square integrable automorphic forms on

$$N_P(\mathsf{A})M_P(\mathsf{Q})A_P(\mathsf{R})^0\backslash G(\mathsf{A}).$$

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