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ON FREE KLEINIAN GROUPS

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This paper is primarily an investigation of geometrically finite free Kleinian groups. Our main result is that such groups lie on the boundary of the appropriate space of Schottky groups; in fact we prove that they are accessible points on the boundary. Similar results were obtained by Marden (unpublished) and Abikoff [1], who showed that every regular b-group lies on the boundary of an appropriate (Bers embedded) Teichmüller space. We also develop necessary and sufficient conditions for a locally free Kleinian group to be geometrically finite. Throughout this paper we will use the term Kleinian group to denote a discrete subgroup of $PSL(2;\mathbf{C})$ which acts discontinuously at some point of $\hat{\mathbf{C}} = \mathbf{C} \cup \{\infty\}$; some authors now call such groups Kleinian groups of the second kind.

Let G be a Kleinian group. We will regard G as acting on both $\hat{\mathbf{C}}$, and on hyperbolic 3-space, $\mathbf{H}^3$, which we will consider to be the upper half space: $\{(z,t) \mid z \in \mathbf{C}, t \in \mathbf{R}^+\}$.

A Kleinian group G is geometrically finite if, in its action on $\mathbf{H}^3$, it has a finite sided fundamental polyhedron.

Let $h \in G$ be parabolic, and let H be the cyclic subgroup of G generated by h. We say that h is cusped if there is an open circular disc $B \subset \hat{\mathbf{C}}$ which is precisely invariant under H in G (i.e., $HB = B$, and $g(B) \cap B = \emptyset$ if $g \in G - H$). We say that h is *doubly cusped* if there are two disjoint open circular discs whose union is precisely invariant under H in G. We say that G is *evenly cusped* if every cusped parabolic element of G is in fact doubly cusped. (We will also sometimes say that the fixed point x of a parabolic element is *cusped*; by this we mean that there is a cusped parabolic element with fixed point x.)

We remark that the cyclic subgroup of G generated by a cusped parabolic element is necessarily a maximal cyclic subgroup of G.

One easily sees that if G is geometrically finite then it is evenly cusped. The converse is clearly false.

As usual, we let $\Omega(G) \subset \hat{\mathbf{C}}$ be the set of discontinuity of G (we assume $\Omega(G) \neq \emptyset$). Ahlfors' finiteness theorem [2] asserts that if G is finitely generated, then $\Omega(G)/G$ is a finite union of finite Riemann surfaces (i.e., closed surfaces less a finite number of points), where the projection map is branched over a finite number of points. We say that a Kleinian group is *analytically finite* if it satisfies the conclusion of Ahlfors' finiteness theorem. It is well known that there are Kleinian groups which are analytically finite, but not finitely generated.

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