Vol. 48, No. 4 DUKE MATHEMATICAL JOURNAL © December 1981

SEMICHARACTERS ON CONNECTED LIE GROUPS

NIELS VIGAND PEDERSEN

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Introduction. Let G be a separable locally compact group. One basic question in the representation theory for such a group is whether its left regular representation λ_G generates a semifinite von Neumann algebra. This was proved by Segal ([28]) and Godement ([12]) to be the case when G is *unimodular*. On the other hand, when no restrictions are imposed on G, one can exhibit examples where λ_G generates a purely infinite von Neumann algebra. It was already conjectured by Segal and Godement ([7] Introduction p. 423) that if G is *connected*, then $\lambda_G(G)''$ is still semifinite. An essential step towards the proof of this conjecture was taken by Pukanszky ([23], [24]) who showed it to be true for all connected solvable Lie groups. Let us mention in passing that Pukanszky in fact obtained much more detailed information: he exhibited a faithful, normal, semifinite trace ϕ on $\lambda_G(G)''$ such that the operators $\lambda_G(\varphi)$, $\varphi \in C_c^\infty(G)$ with $\phi(\lambda_G(\varphi)^*\lambda_G(\varphi)) < +\infty$, form a weakly dense subset in $\lambda_G(G)''$. In particular λ_G paired with this trace is a traceclass representation ([6] 6.6.1 Définition). Not long after this the validity of the conjecture for a general connected G was proved by Dixmier ([7]) and Pukanszky ([25]). At this point let us remark that the existence of a faithful, normal, semifinite trace on $\lambda_G(G)''$ together with which λ_G is traceclass remains open in general.

The purpose of the present paper is to discuss an "infinitesimal" analog of the type of questions considered above. The problem is naturally formulated using the notion of a semitrace on a locally compact group. Let us recall that in [20] we defined a semitrace on a locally compact group G to be a certain weight f on the

Received November 21, 1980. Revision received June 10, 1981. Supported by a fellowship from the University of Copenhagen and by Det Naturvidenskabelige Forskningsråd, Denmark. A part of this work was performed while the author was a visitor at the Department of Mathematics, University of California, Berkeley, USA.