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COMPLEX MANIFOLDS IN PSEUDOCONVEX BOUNDARIES

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We consider a pseudoconvex domain $\Omega \subset \mathbf{C}^n$ with smooth boundary, and we let $M \subset \partial\Omega$ be a smooth manifold. For convenience we take "smooth" to mean C^∞, although it is clear that the arguments remain valid in the category of finite differentiability. Let TM denote the tangent bundle of M, and let HM be the "holomorphic" tangent bundle, i.e., HM_p is the maximal complex linear subspace of TM_p under the natural inclusion $TM_p \subset \mathbf{C}^n$ for all $p \in M$. We assume that M is CR, which is to say that $\dim_{\mathbf{R}} HM_p = 2m$ is locally constant for $p \in M$, and for $p \in M \cup \partial M$ if M is a smooth manifold-with-boundary.

Let us write

$$Q = \{(\tau,\sigma) \in \mathbf{R}^+ \times \mathbf{C} : |\sigma|^2 = \tau\}$$

$$\tilde{Q} = \{(\tau,\sigma) \in \mathbf{R}^+ \times \mathbf{C} : |\sigma|^2 \leqslant \tau\}.$$

We will consider a *smooth construction of disks at p with boundaries in M*, by which we mean a mapping

$$F : U(\epsilon) \cap (\mathbf{R}^l \times \tilde{Q}) \to \mathbf{C}^n$$

such that

$$F(0) = p \tag{1a}$$

$$F\big(U(\epsilon) \cap (\mathbf{R}^l \times Q)\big) \subset M \tag{1b}$$

F extends to a C^∞ diffeomorphism of

$$U(\epsilon) \cap (\mathbf{R}^l \times \tilde{Q}) \cap \{\tau > 0\}, \quad \text{and} \quad F \in C^\infty\big(U(\epsilon) \cup \mathbf{R}^l \times Q\big) \cap C\big(U(\epsilon) \cap (\mathbf{R}^l \times \tilde{Q})\big) \tag{1c}$$

$$F \quad \text{is holomorphic on the complex disks of} \quad \tilde{Q} \tag{1d}$$

with the notation

$$\begin{aligned} &M_0 = M \cap F(\{\tau = 0\} \cap U(\epsilon)),\\ &\tilde{M} = F\big(U(\epsilon) \cap (\mathbf{R}^l \times \tilde{Q})\big),\\ &M' = \tilde{M} \setminus M, \quad \text{we have that}\\ &\tilde{M} \setminus M_0 \quad \text{is a smooth } CR \text{ manifold with boundary } M \setminus M_0, \quad \text{and}\\ &\dim_{\mathbf{R}} HM' = 2m + 2. \end{aligned} \tag{1e}$$

where $U(\epsilon)$ denotes an open ϵ-neighborhood of the origin, and $l = \dim TM_p - 2$.

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